ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ

Εισαγωγή στον Προγραμματισμό
Introduction to Programming

Διάλεξη 14.β: Συναρτήσεις και Γραφικά

Γ. Παπαγιαννάκης

ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ & ΘΡΗΣΚΕΥΜΑΤΩΝ, ΠΟΛΙΤΙΣΜΟΥ & ΑΘΛΗΤΙΣΜΟΥ
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Ευρωπαϊκή Ένωση
Ευρωπαϊκό Κοινωνικό Ταμείο
ΕΣΠΑ 2007-2013
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

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HY-150 Προγραμματισμός
CS-150 Programming

Lecture 14b:
Functions and graphing

G. Papagiannakis

Abstract

- Here we present ways of graphing functions and data and some of the programming techniques needed to do so, notably scaling.

Note

- This course is about programming
  - The examples – such as graphics – are simply examples of
    - Useful programming techniques
    - Useful tools for constructing real programs

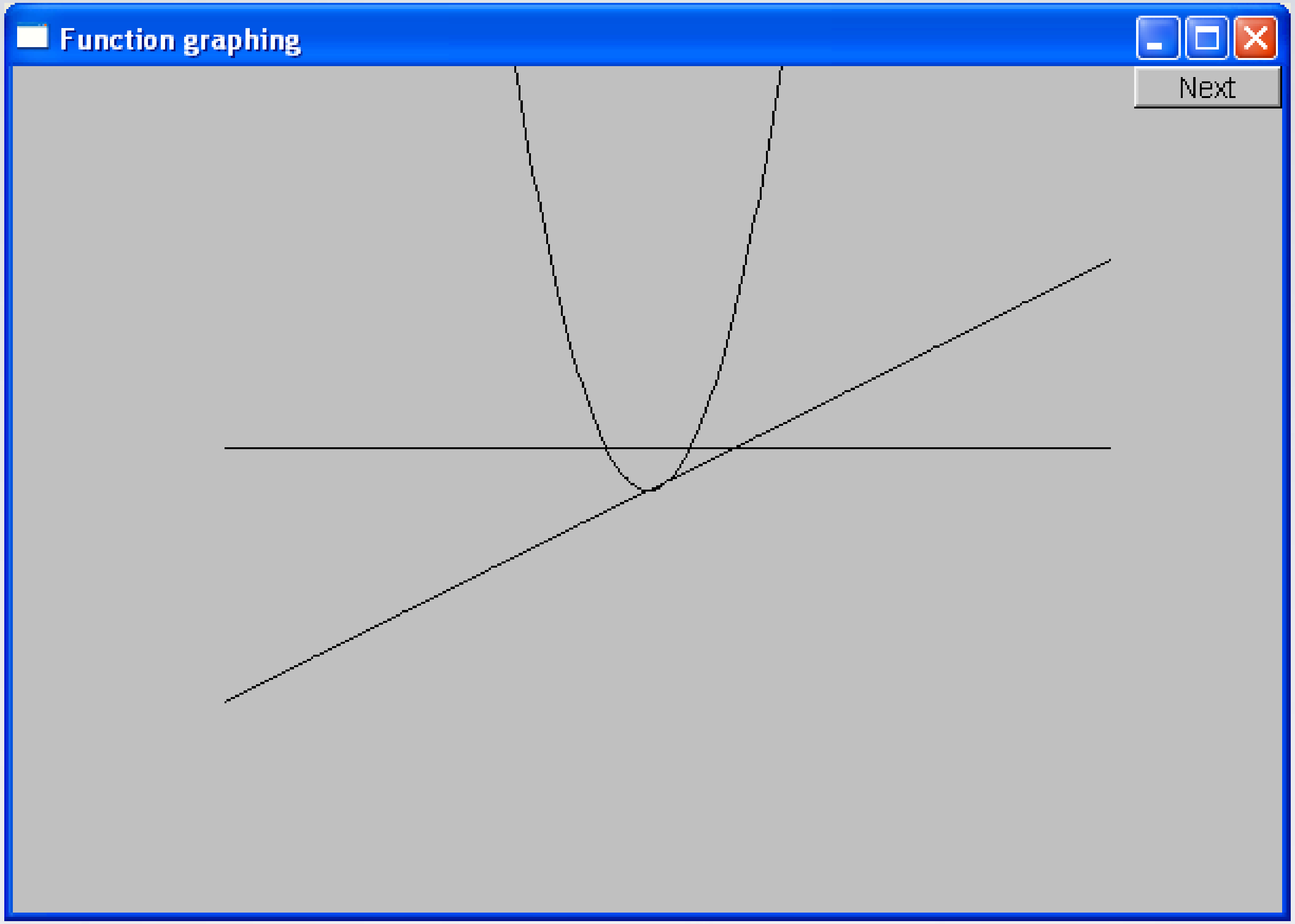
    Look for the way the examples are constructed
  - How are "big problems" broken down into little ones and solved separately?
  - How are classes defined and used
    - Do they have sensible data members?
    - Do they have useful member functions?
  - Use of variables
    - Are there too few?
    - Too many?
    - How would you have named them better?

Graphing functions

- Start with something really simple
  - Always remember “Hello, World!”
- We graph functions of one argument yielding a one value
  - Plot (x, f(x)) for values of x in some range [r1,r2)
- Let’s graph three simple functions

```
double one(double x) { return 1; }            // y==1
double slope(double x) { return x/2; }        // y==x/2
double square(double x) { return x*x; }       // y==x*x
```

Functions




```
double one(double x) { return 1; }              // y==1
double slope(double x) { return x/2; }          // y==x/2
double square(double x) { return x*x; }         // y==x*x
```

How do we write code to do this?

Function to be graphed

```
Simple_window win0(Point(100,100),xmax,ymax,"Function graphing");

Function s(one,-10,11,orig,n_points,x_scale,y_scale);
Function s2(slope,-10,11,orig,n_points,x_scale,y_scale);
Function s3(square,-10,11,orig,n_points,x_scale,y_scale);

win0.attach(s);
win0.attach(s2);
win0.attach(s3);

win0.wait_for_button( );
```

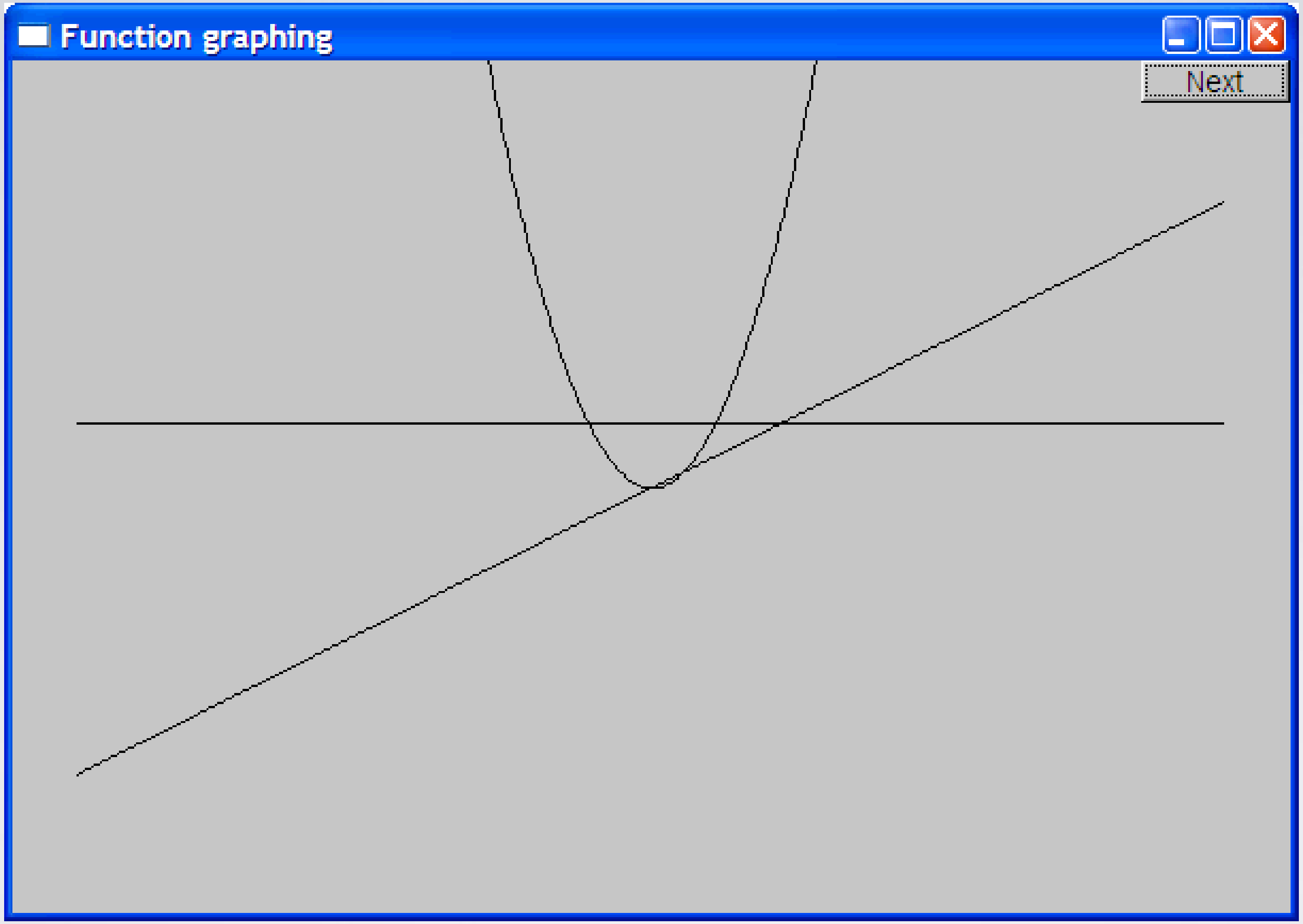
Range in which to graph

“stuff” to make the graph fit into the window

We need some Constants

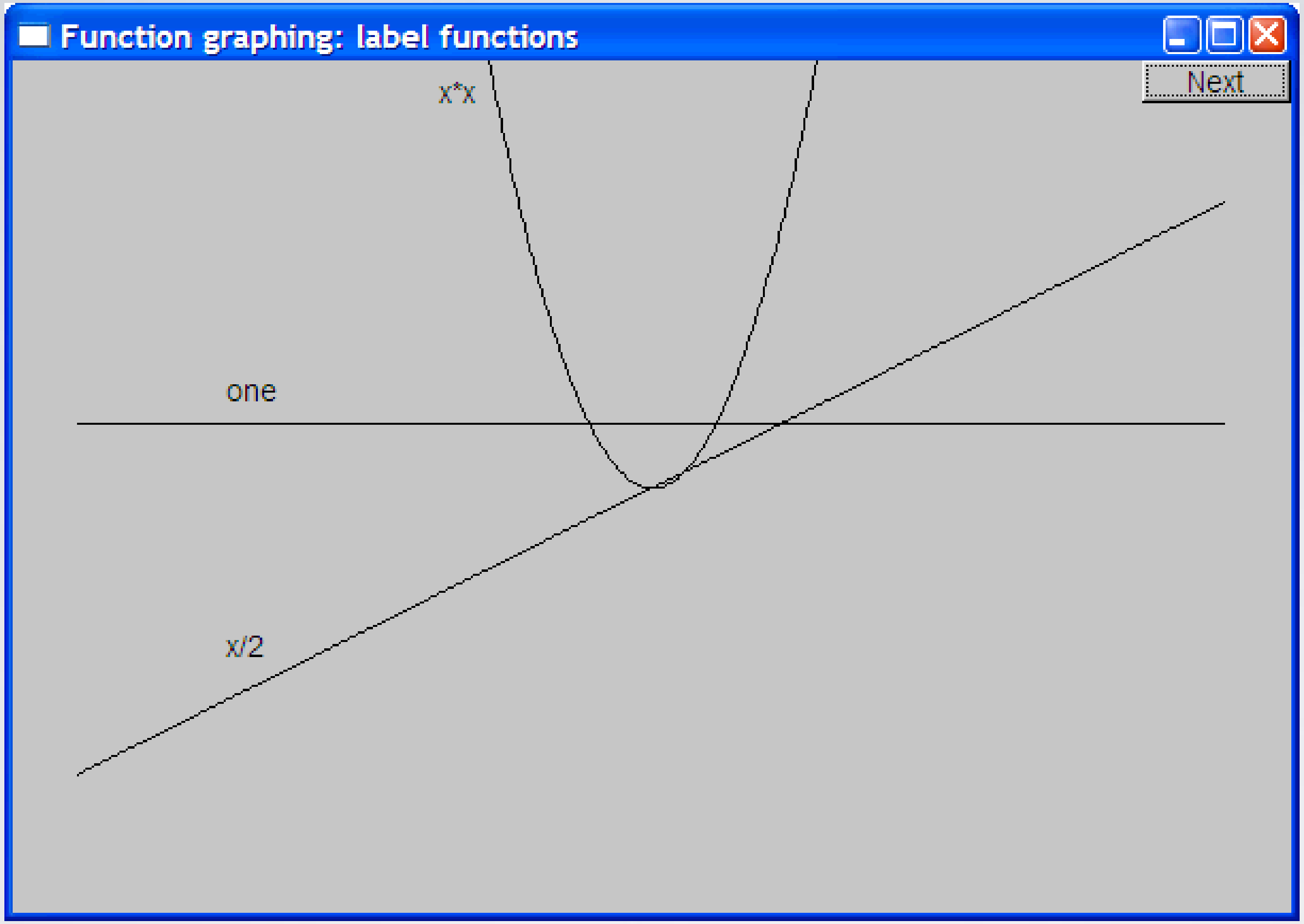
```
const int xmax = 600;                       // window size
const int ymax = 400;

const int x_orig = xmax/2;
const int y_orig = ymax/2;
const Point orig(x_orig, y_orig);   // position of (0,0) in window

const int r_min = -10;                      // range [-10:11) == [-10:10] of x
const int r_max = 11;

const int n_points = 400;                   // number of points used in range

const int x_scale = 20;                     // scaling  factors
const int y_scale = 20;

// Choosing a center (0,0), scales, and number of points can be fiddly
// The range usually comes from the definition of what you are doing
```

Functions – but what does it mean?



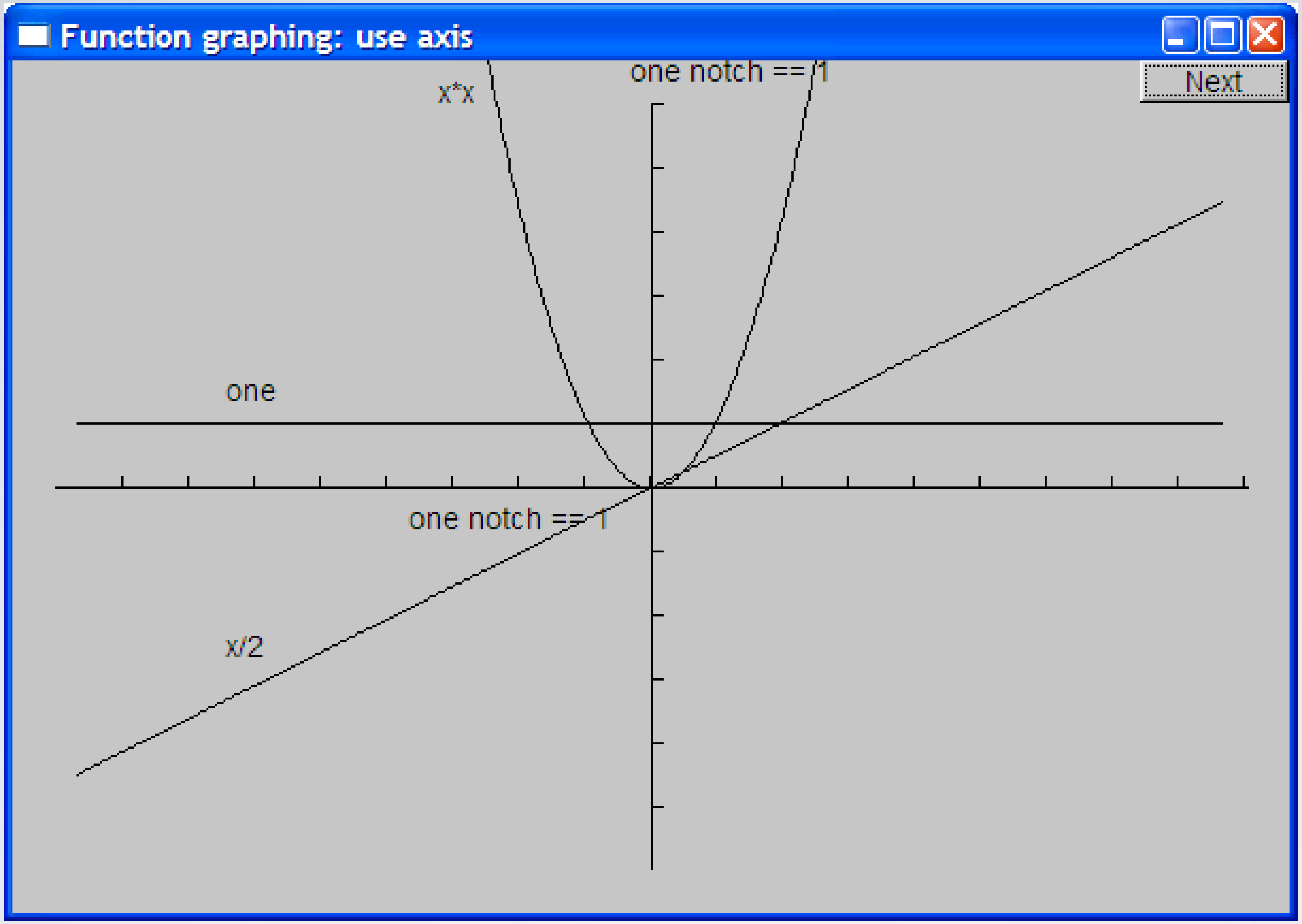
- What's wrong with this?
- No axes (no scale)
- No labels

Label the functions




```
Text ts(Point(100,y_orig-30),"one");
Text ts2(Point(100,y_orig+y_orig/2-10),"x/2");
Text ts3(Point(x_orig-90,20),"x*x");
```

Add x-axis and y-axis

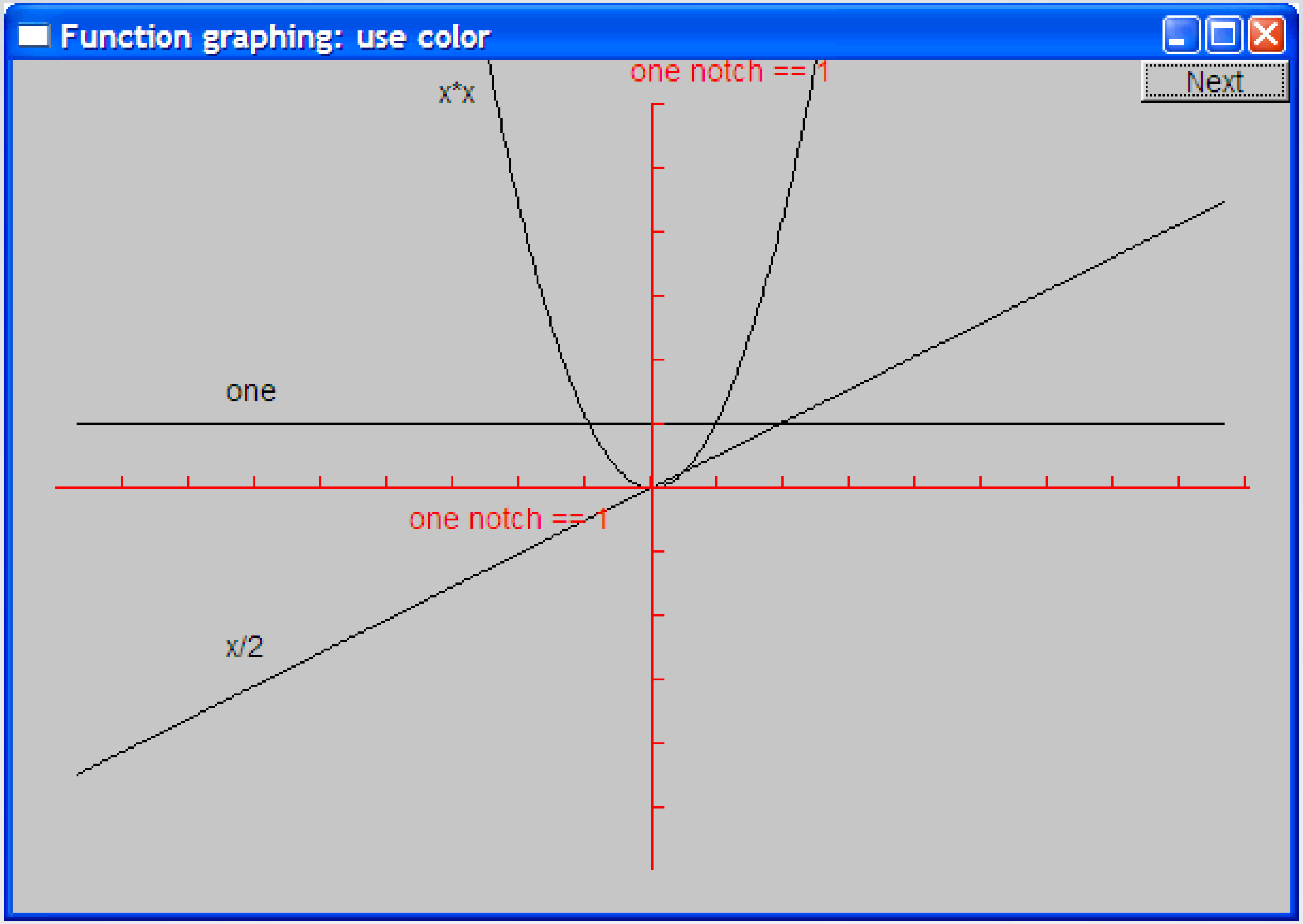


- We can use axes to show (0,0) and the scale

Axis x(Axis::x, Point(20,y_orig), xlength/x_scale, "one notch == 1 ");

Axis y(Axis::y, Point(x_orig, ylength+20, ylength/y_scale, "one notch == 1");

Use color (in moderation)



s.set_color(Color::green); **x.set_color(Color::red);**

y.set_color(Color::red); **ts.set_color(Color::green);**

The implementation of Function

- We need a type for the argument specifying the function to graph
  - **typedef** can be used to declare a new name for a type
    - **typedef int Color;** *// now Color means int*

  - Define the type of our desired argument, **Fct**
    - **typedef double Fct(double);** *// now Fct means function*
      // taking a double argument
      // and returning a double

  - Examples of functions of type **Fct**:

```
double one(double x) { return 1; }          // y==1
double slope(double x) { return x/2; }      // y==x/2
double square(double x) { return x*x; }     // y==x*x
```

Now Define "Function"

```
struct Function : Shape                    // Function is derived from Shape
{
   // all it needs is a constructor:
   Function(
      Fct f,                        // f is a Fct  (takes a double, returns a double)

      double r1,    // the range of x values (arguments to f) [r1:r2)
      double r2,
      Point orig,   // the screen location of (0,0)
      int count,                    // number of points used to draw the function
                                    // (number of line segments used is count-1)

      double xscale ,               // the location (x,f(x)) is (xscale*x,yscale*f(x))
      double yscale
   );
};
```

Implementation of Function

```
Function::Function( Fct f,
                    double r1, double r2,   // range
                    Point xy,
                    int count,
                    double xscale, double yscale )
{
    if (r2-r1<=0) error("bad graphing range");
    if (count <=0) error("non-positive graphing count");
    double dist = (r2-r1)/count;
    double r = r1;
    for (int i = 0; i<count; ++i) {
            add(Point(xy.x+int(r*xscale), xy.y-int(f(r)*yscale)));
            r += dist;
    }
}
```

Default arguments

- Seven arguments are too many!
  - Many too many
  - We're just asking for confusion and errors
  - Provide defaults for some (trailing) arguments
    - Default arguments are often useful for constructors

```
struct Function : Shape {
   Function( Fct f, double r1, double r2, Point xy,
            int count = 100, double xscale = 25, double yscale=25 );
};

Function f1(sqrt, 0, 11, orig, 100, 25, 25 );  // ok (obviously)
Function f2(sqrt, 0, 11, orig, 100, 25);       // ok: exactly the same as f1
Function f3(sqrt, 0, 11, orig, 100);           // ok: exactly the same as f1
Function f4(sqrt, 0, 11, orig);                // ok: exactly the same as f1
```

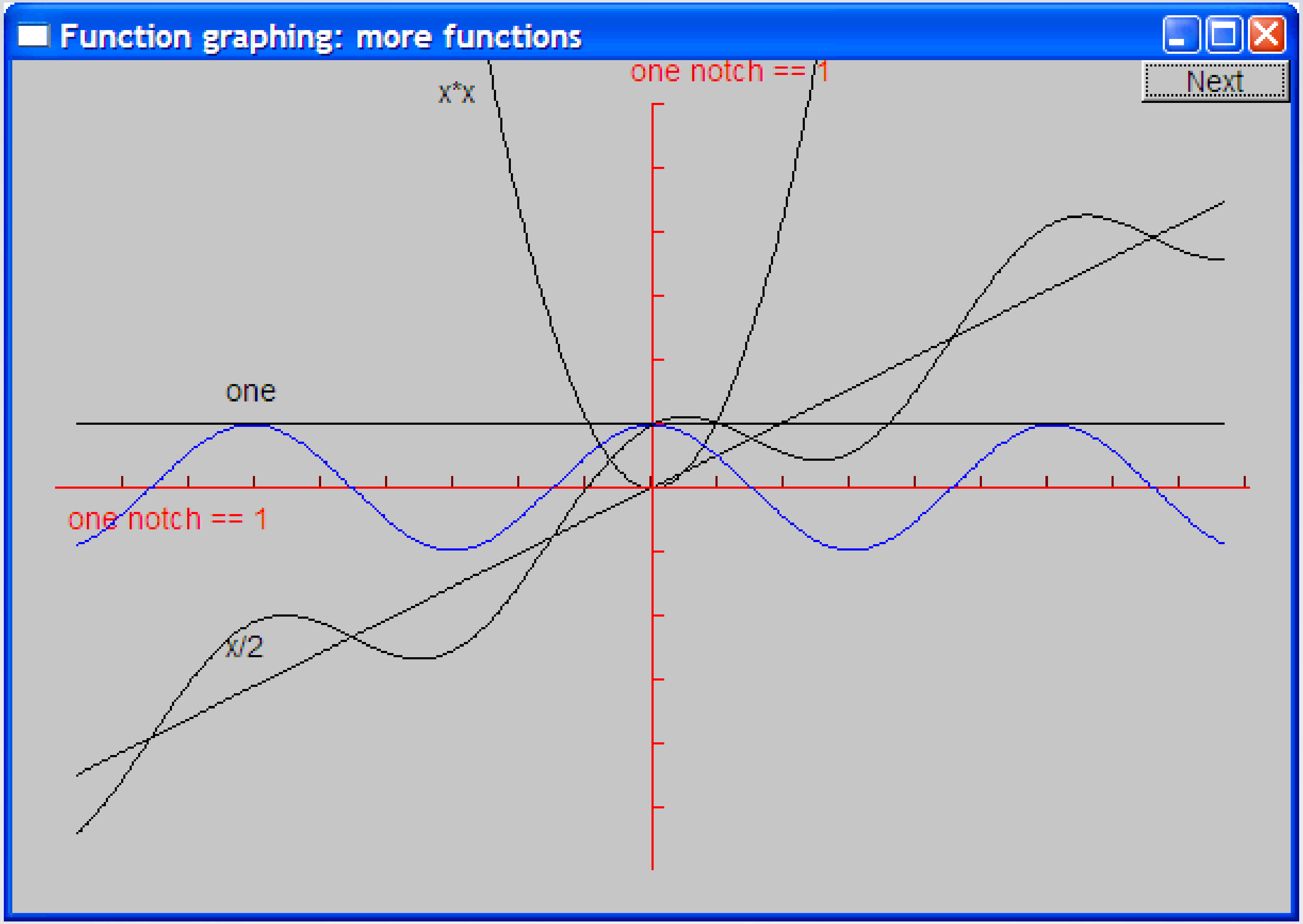
Function

- Is **Function** a “pretty class”?
  - No
    - Why not?
  - What could you do with all of those position and scaling arguments?
    - See 15.6.3 for one minor idea
  - If you can’t do something genuinely clever, do something simple, so that the user can do anything needed
    - Such as adding parameters so that the caller can control

Some more functions

```
#include<cmath>        // standard mathematical functions


// You can combine functions (e.g., by addition):
double sloping_cos(double x) { return cos(x)+slope(x); }


Function s4(cos,-10,11,orig,400,20,20);
s4.set_color(Color::blue);
Function s5(sloping_cos,-10,11,orig,400,20,20);
```

Cos and sloping-cos



Standard mathematical functions (<cmath>)

- **double abs(double);** *// absolute value*

- **double ceil(double d);** *// smallest integer* **>= d**
- **double floor(double d);** *// largest integer* **<= d**

- **double sqrt(double d);** *//* ***d*** *must be non-negative*

- **double cos(double);**
- **double sin(double);**
- **double tan(double);**
- **double acos(double);** *// result is non-negative; "a" for "arc"*
- **double asin(double);** *// result nearest to 0 returned*
- **double atan(double);**
- **double sinh(double);** *// "h" for "hyperbolic"*
- **double cosh(double);**
- **double tanh(double);**

Standard mathematical functions (<cmath>)

- **double exp(double);** *// base e*
- **double log(double d);** *// natural logarithm (base e)* **; d** *must be positive*
- **double log10(double);** *// base 10 logarithm*

- **double pow(double x, double y);** *//* ***x*** *to the power of* ***y***
- **double pow(double x, int y);** *//* ***x*** *to the power of* ***y***
- **double atan2(double x, double y);** *// atan(x/y)*
- **double fmod(double d, double m);** *// floating-point remainder*
  // same sign as d%m
- **double ldexp(double d, int i);** *// d*pow(2,i)*

Why graphing?

- Because you can see things in a graph that are not obvious from a set of numbers
  - How would you understand a sine curve if you couldn’t (ever) see one?
- Visualization is
  - key to understanding in many fields
  - Used in most research and business areas
    - Science, medicine, business, telecommunications, control of large systems

An example: e^x

e^x == **1**

+ x

+ x^2/2!

+ x^3/3!

+ x^4/4!

+ x^5/5!

+ x^6/6!

+ x^7/7!

+ …

Where ! Means factorial (e.g. 4!==4*3*2*1)

Simple algorithm to approximate e^x

```
double fac(int n) { /* … */ }          // factorial

double term(double x, int n)           // x^n/n!
{
   return pow(x,n)/fac(n);
}

double expe(double x, int n)           // sum of n terms of x
{
   double sum = 0;
   for (int i = 0; i<n; ++i) sum+=term(x,i);
   return sum;
}
```

Simple algorithm to approximate e^x

- But we can only graph functions of one argument, so how can we get graph **expe(x,n)** for various **n**?

```
int expN_number_of_terms = 6;        // nasty sneaky argument to expN

double expN(double x)            // sum of expN_number_of_terms terms of x
{
   return expe(x,expN_number_of_terms);
}
```

“Animate” approximations to e^x

```
Simple_window win(Point(100,100),xmax,ymax,"");
// the real exponential :
Function real_exp(exp,r_min,r_max,orig,200,x_scale,y_scale);
real_exp.set_color(Color::blue);
win.attach(real_exp);

const int xlength = xmax-40;
const int ylength = ymax-40;
Axis x(Axis::x, Point(20,y_orig),
        xlength, xlength/x_scale, "one notch == 1");
Axis y(Axis::y, Point(x_orig,ylength+20),
        ylength, ylength/y_scale, "one notch == 1");

win.attach(x);
win.attach(y);
x.set_color(Color::red);
y.set_color(Color::red);
```

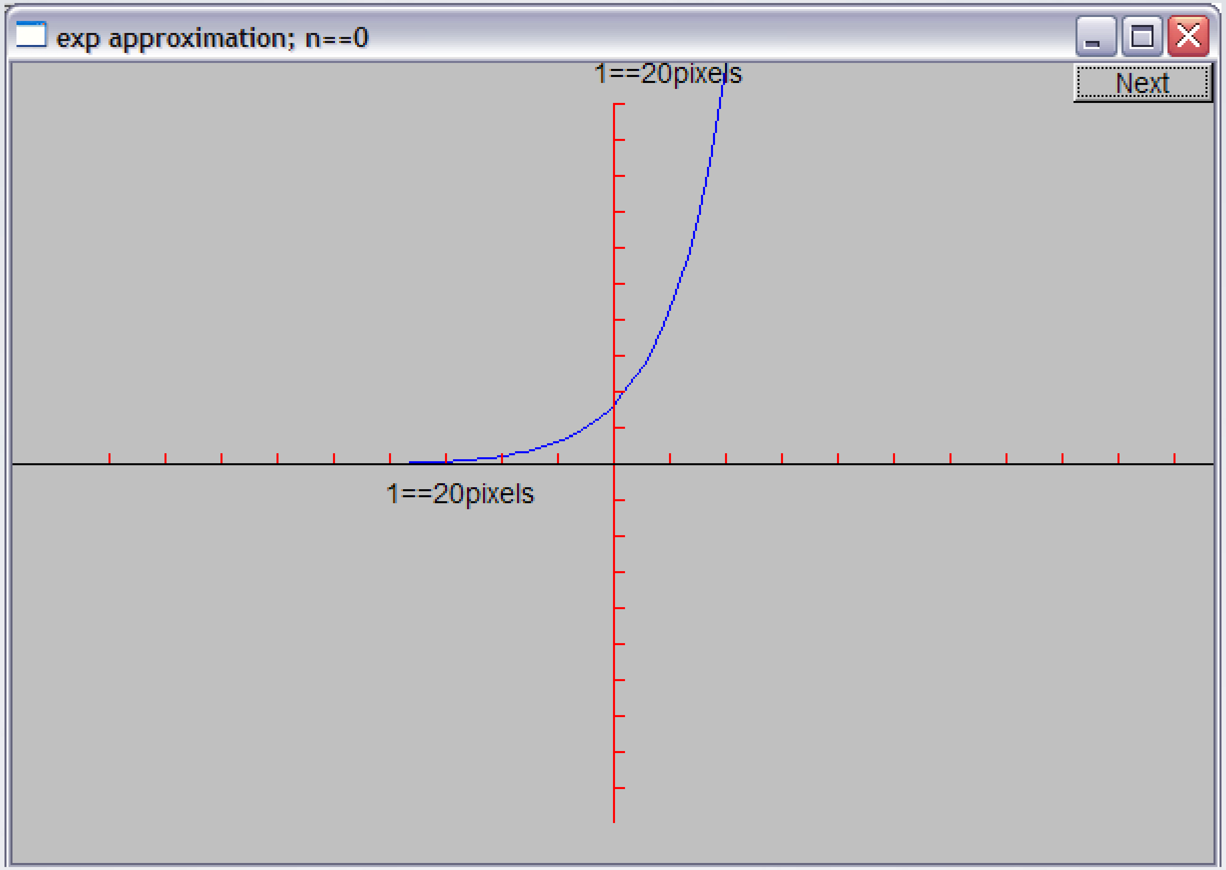
"Animate" approximations to e^x

```
for (int n = 0; n<50; ++n) {
    ostringstream ss;
    ss << "exp approximation; n==" << n ;
    win.set_label(ss.str().c_str());
    expN_number_of_terms = n;    // nasty sneaky argument to expN

    // next approximation:
    Function e(expN,r_min,r_max,orig,200,x_scale,y_scale);

    win.attach(e);
    wait_for_button();    // give the user time to look
    win.detach(e);
}
```

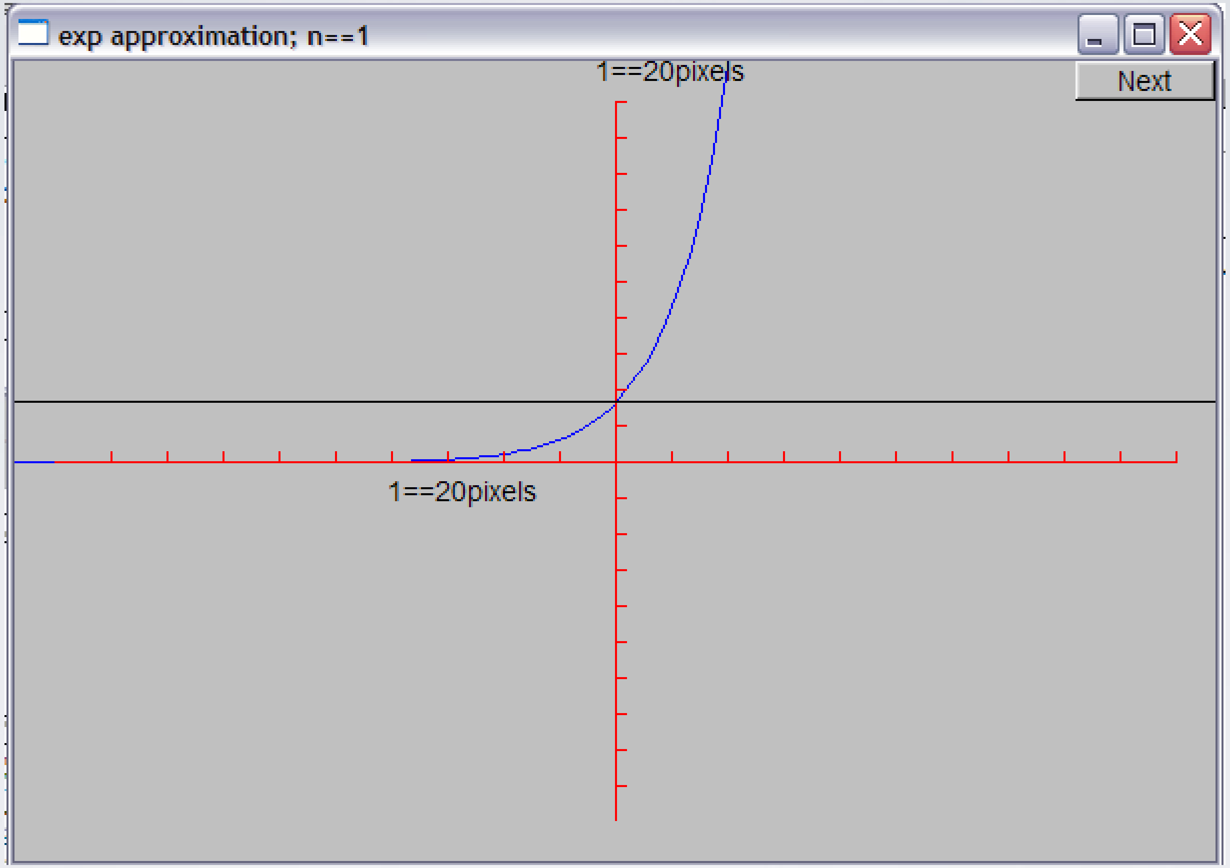
Demo

- The following screenshots are of the successive approximations of **exp(x)** using **expe(x,n)**

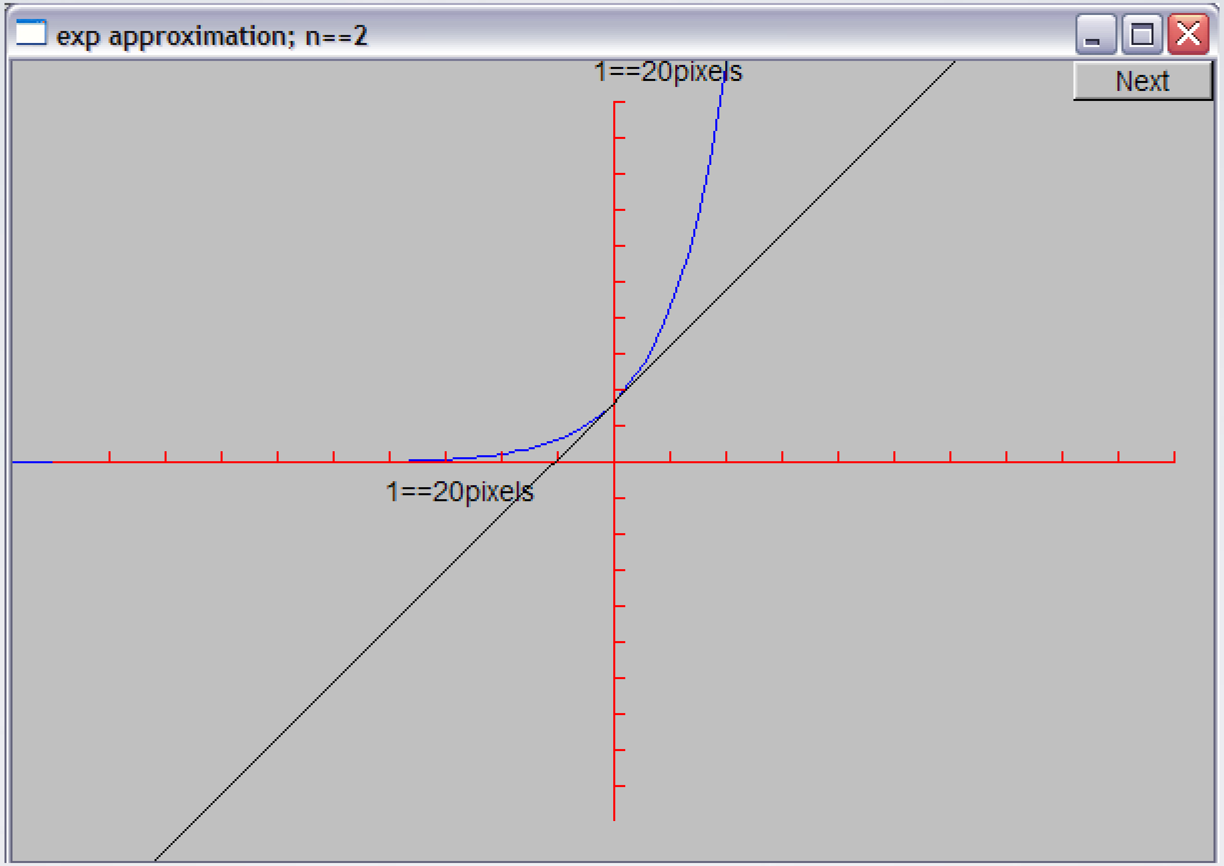
Demo n = 0



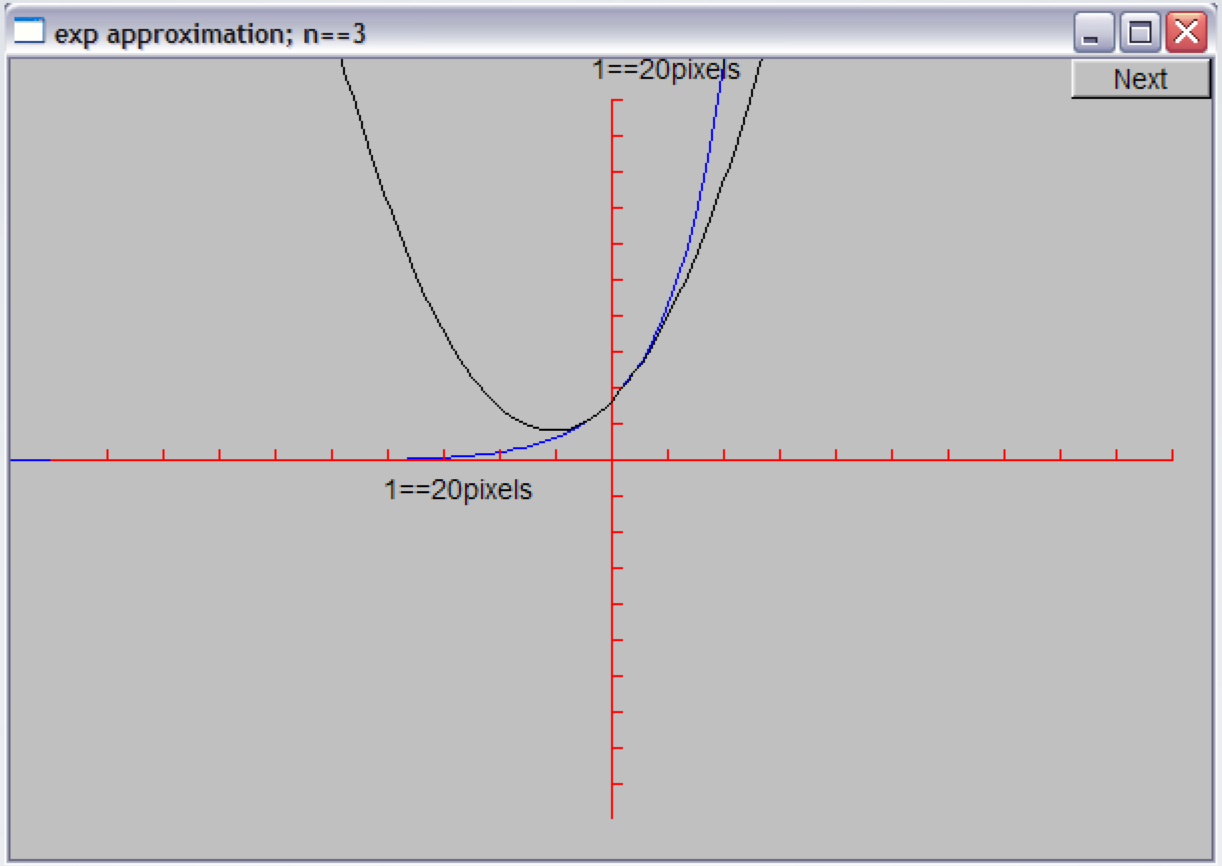
Demo n = 1



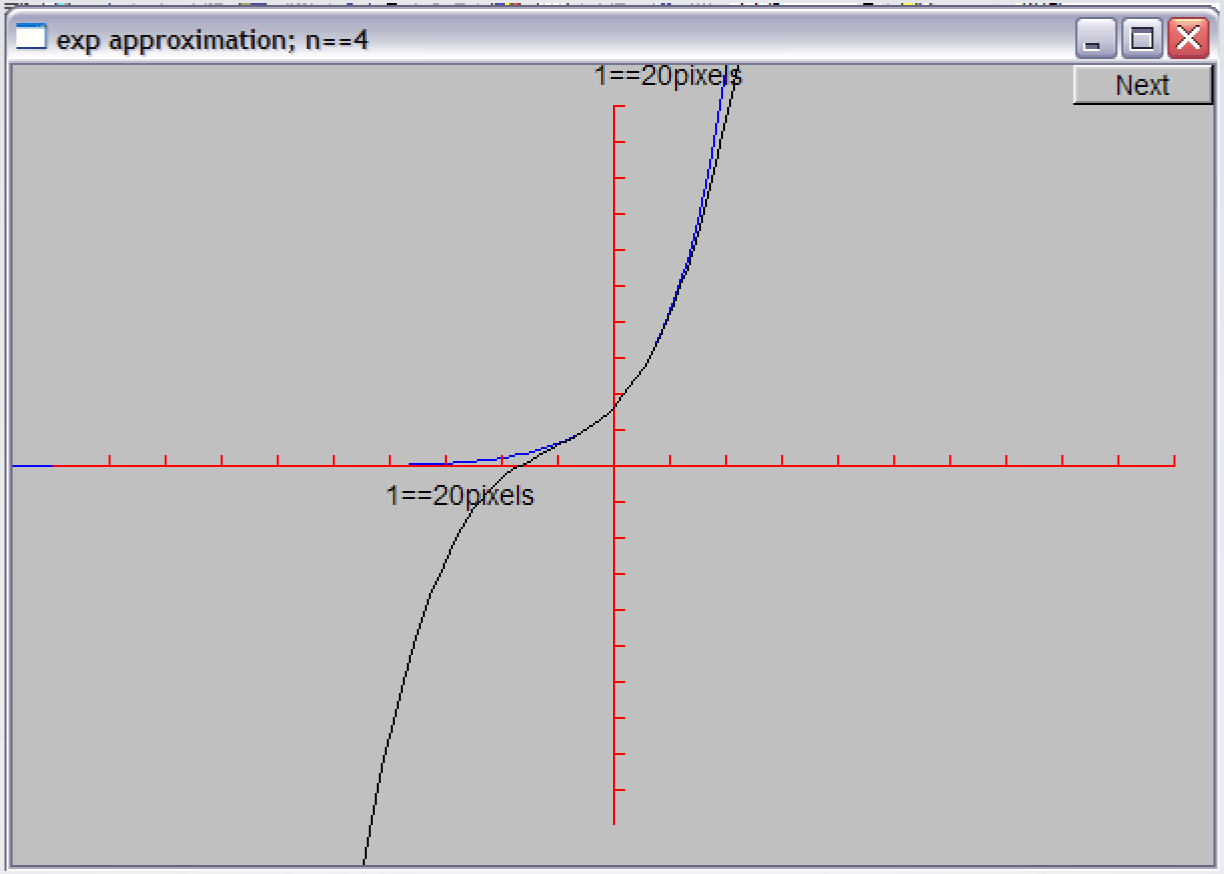
Demo n = 2



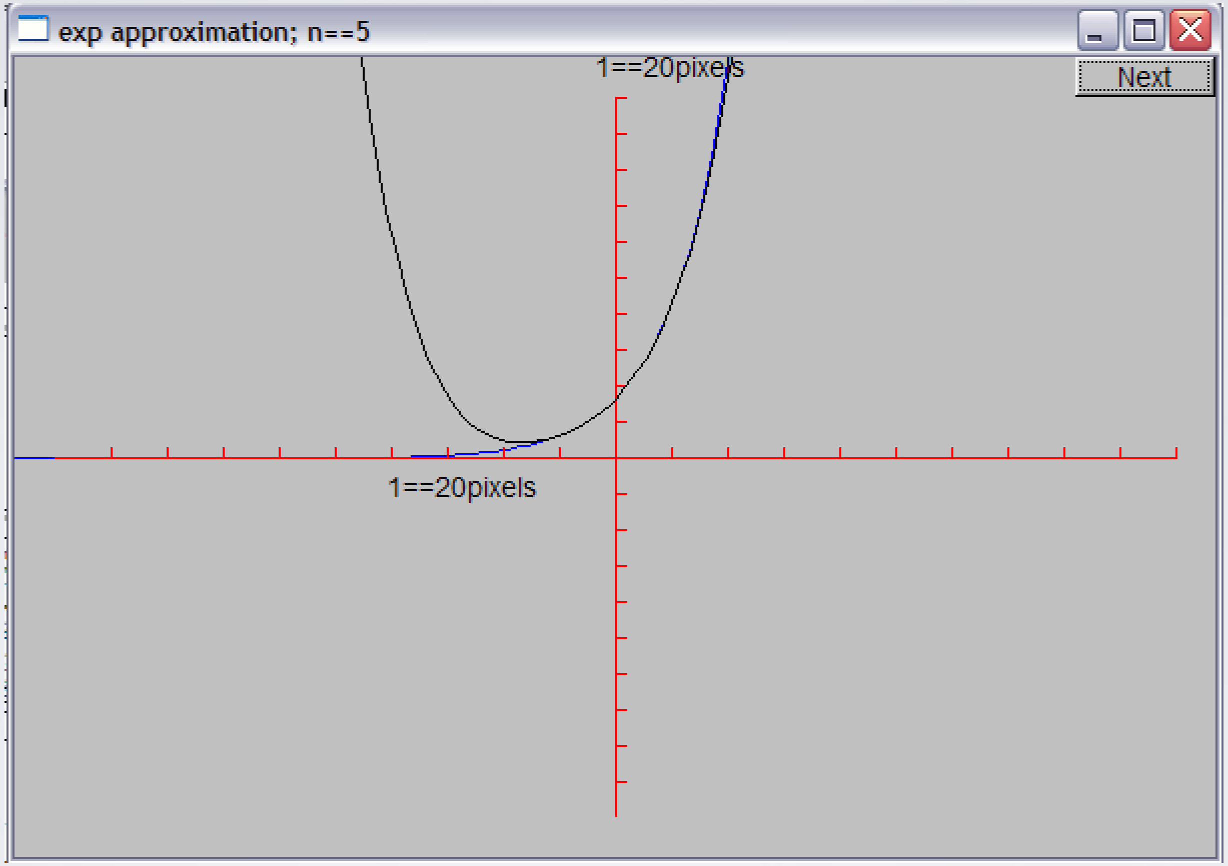
Demo n = 3



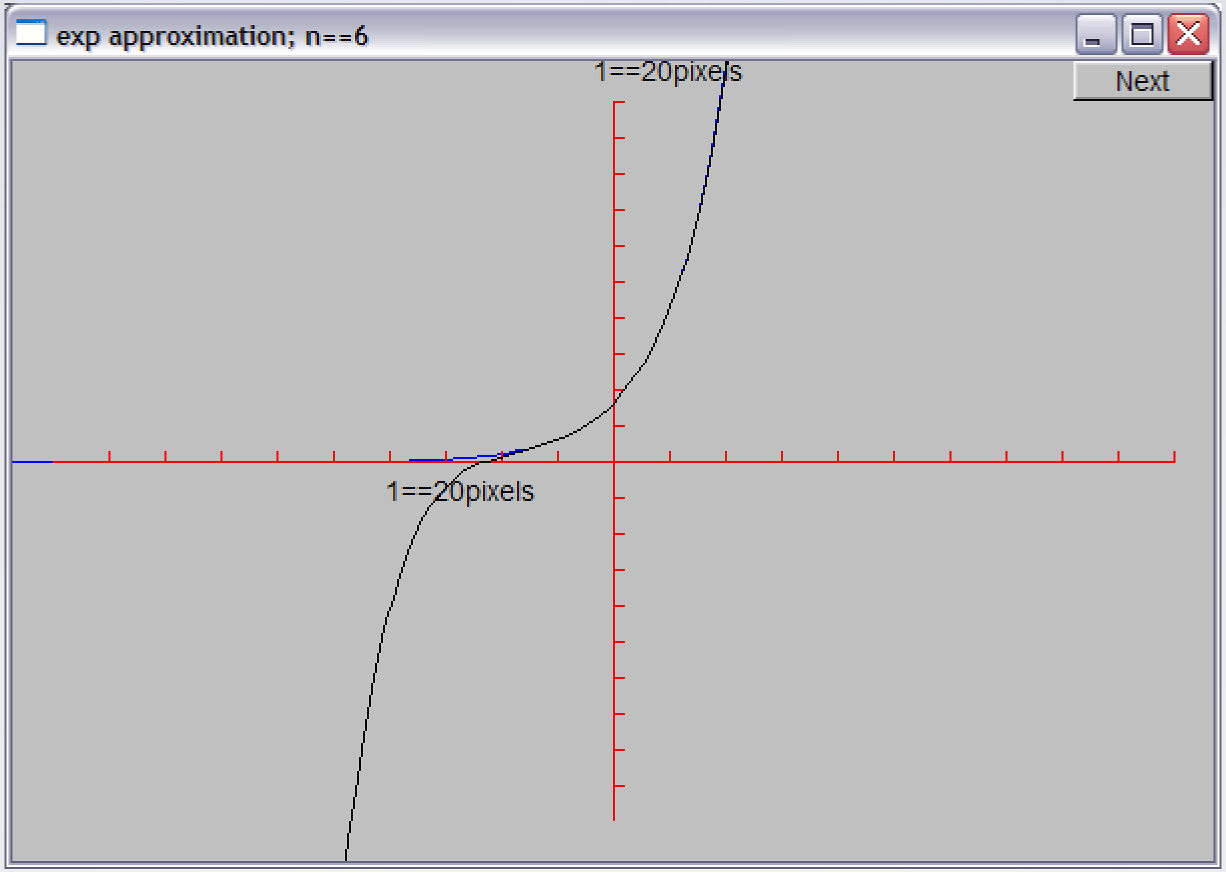
Demo n = 4



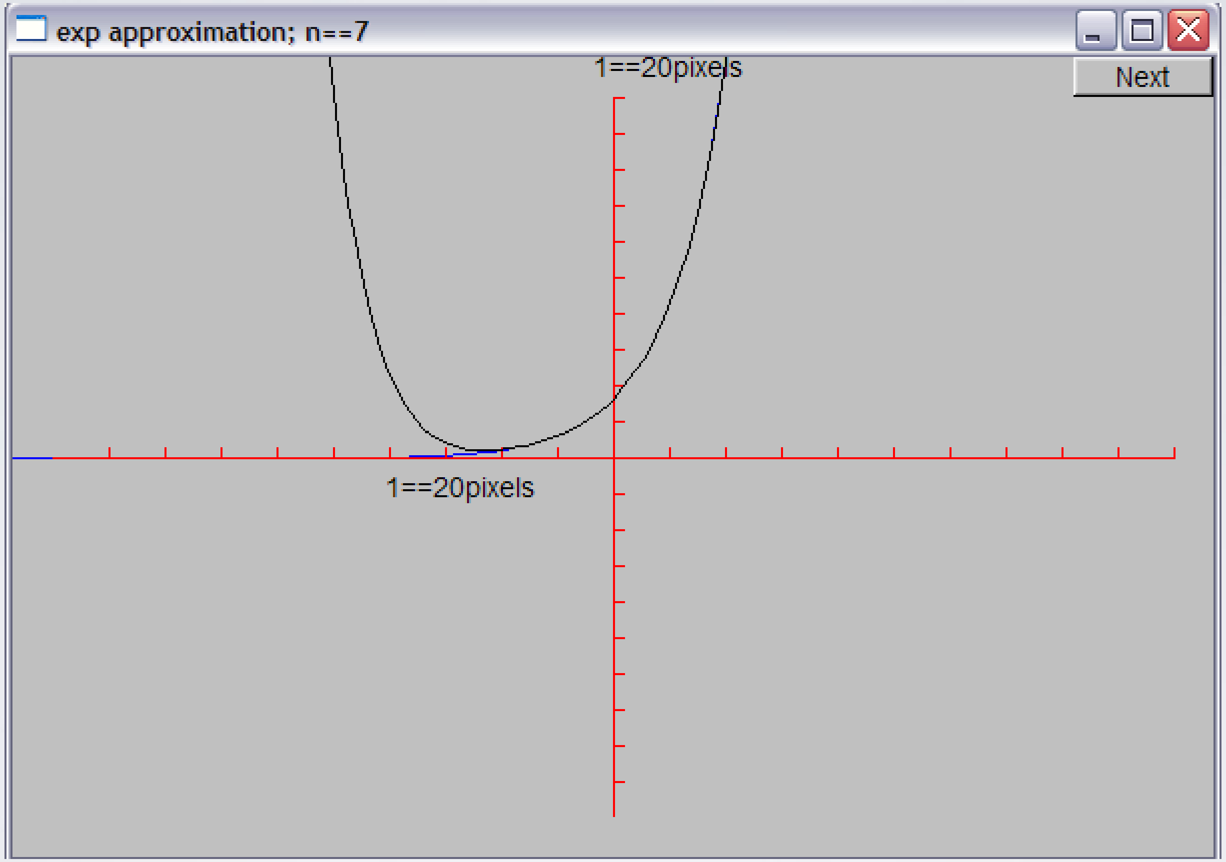
Demo n = 5



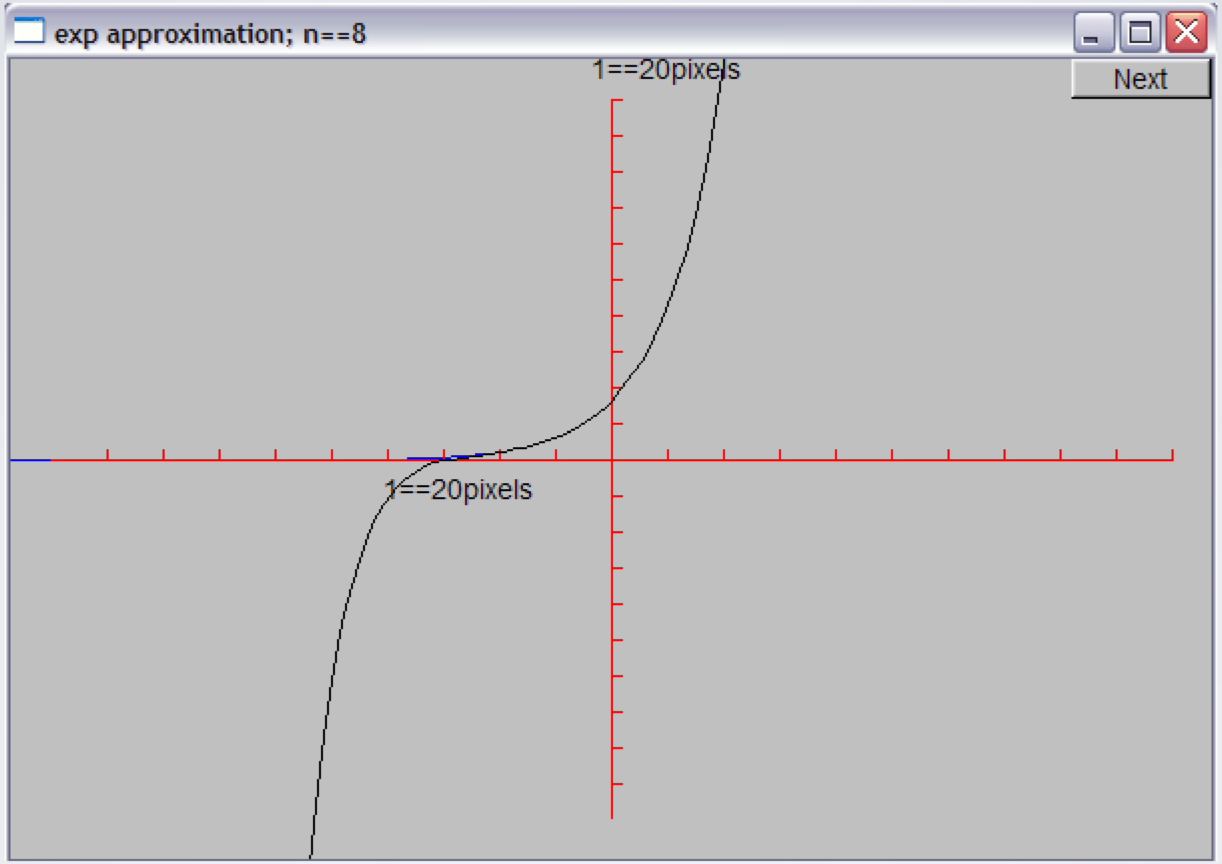
Demo n = 6



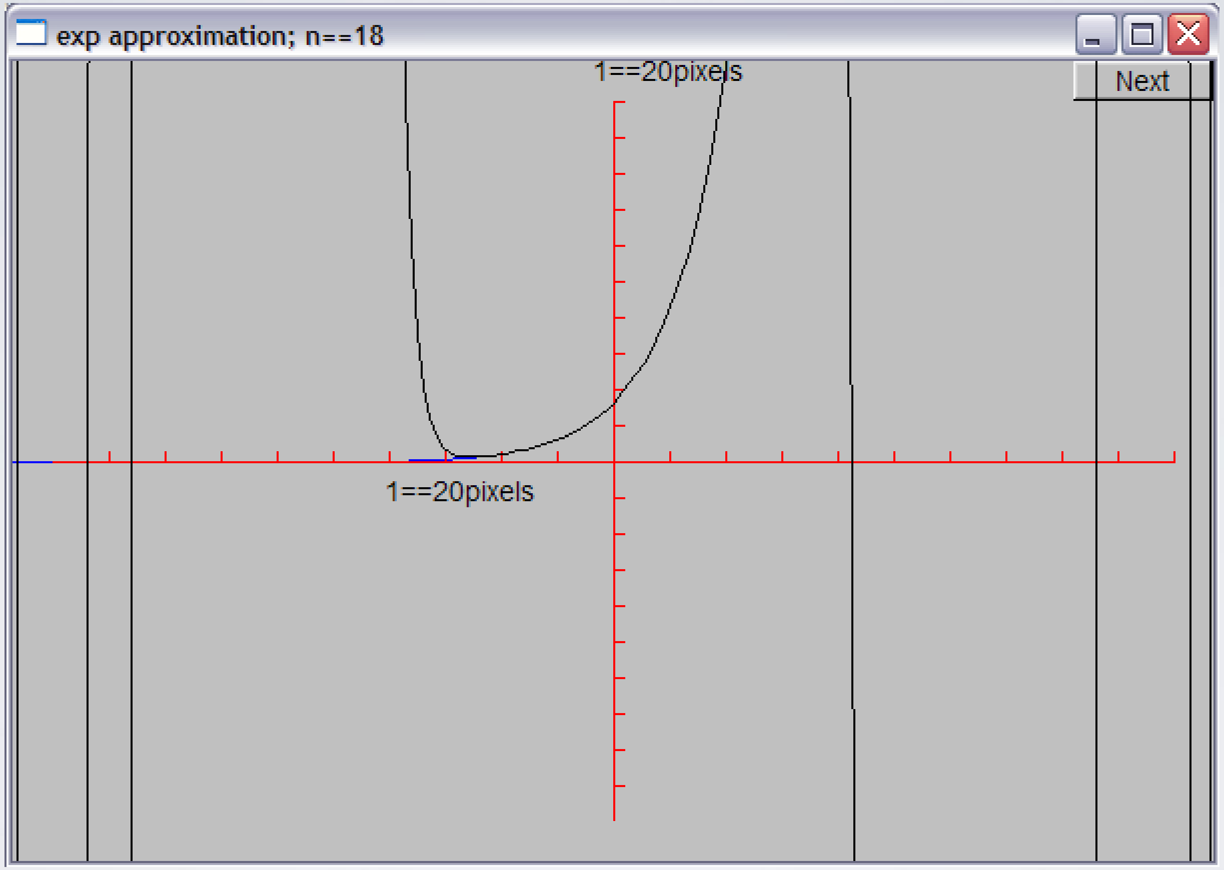
Demo n = 7



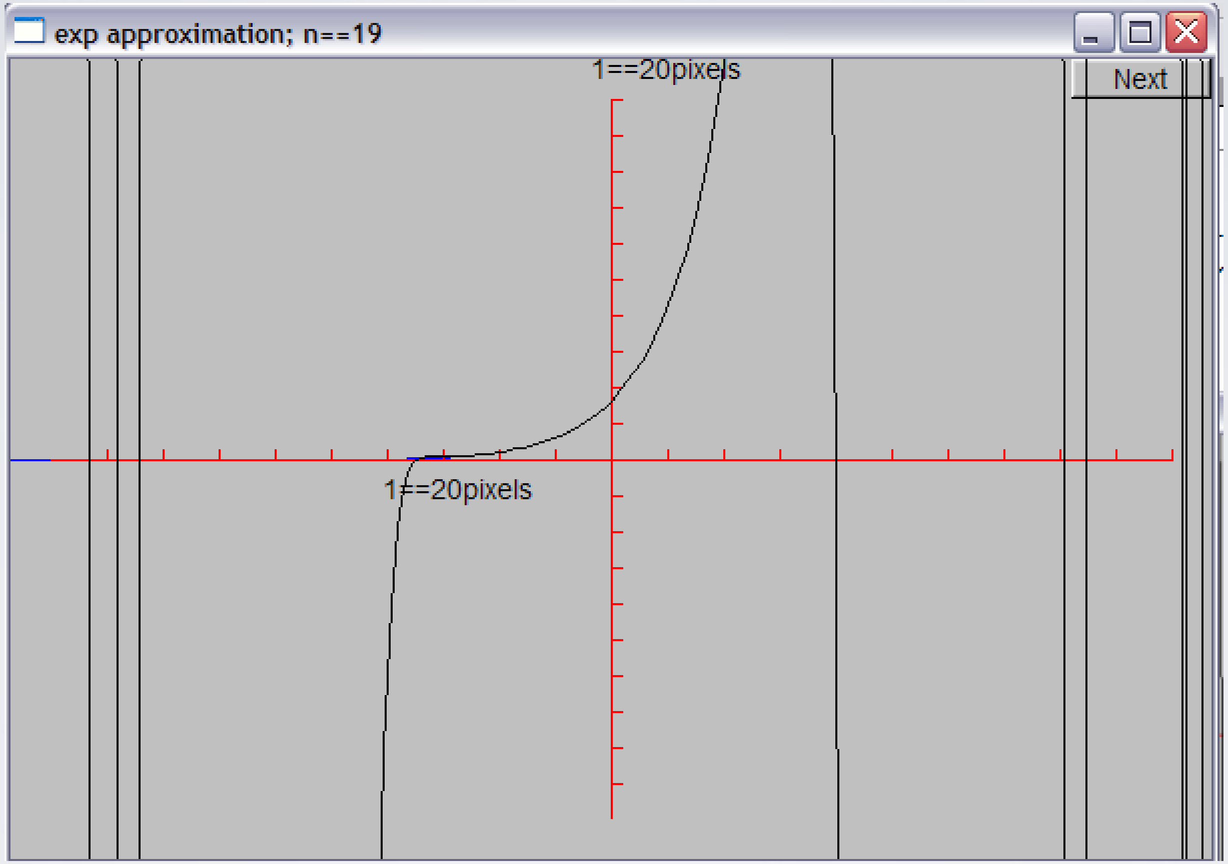
Demo n = 8



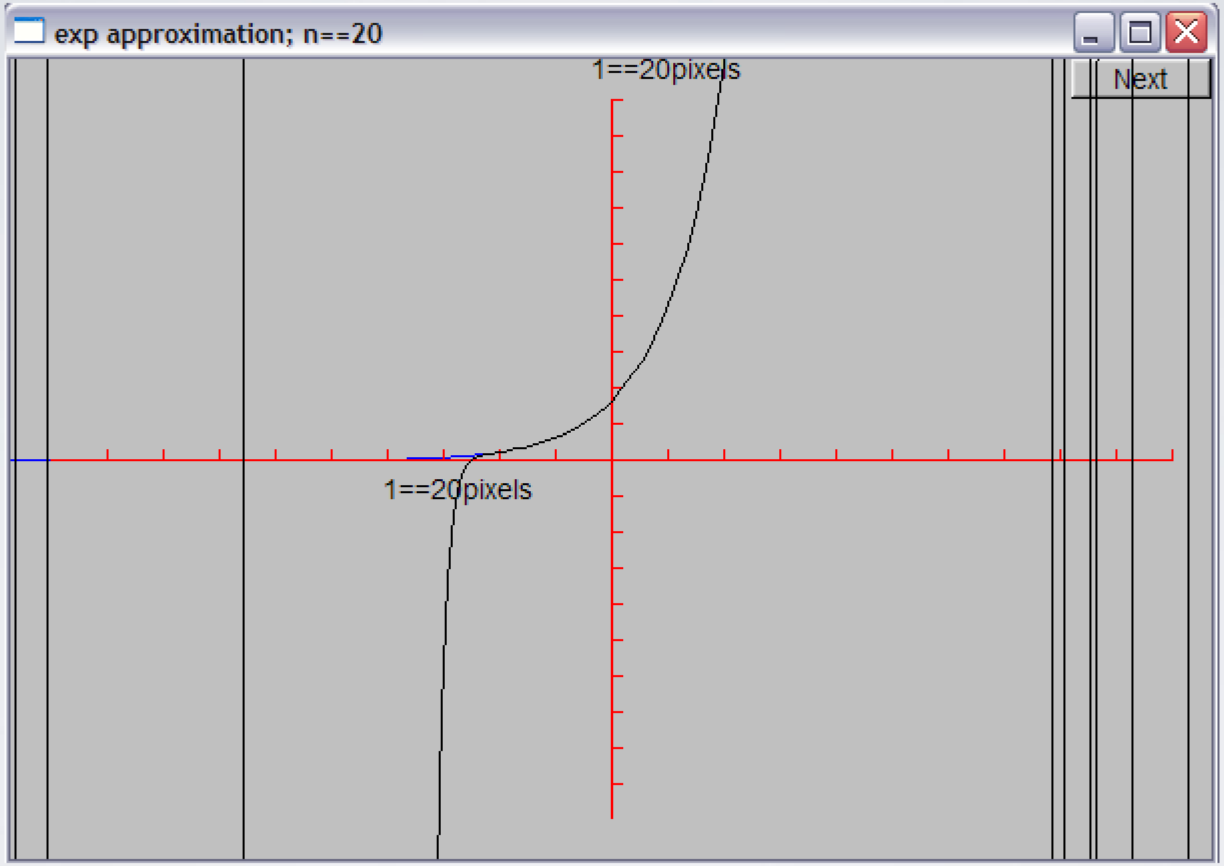
Demo n = 18



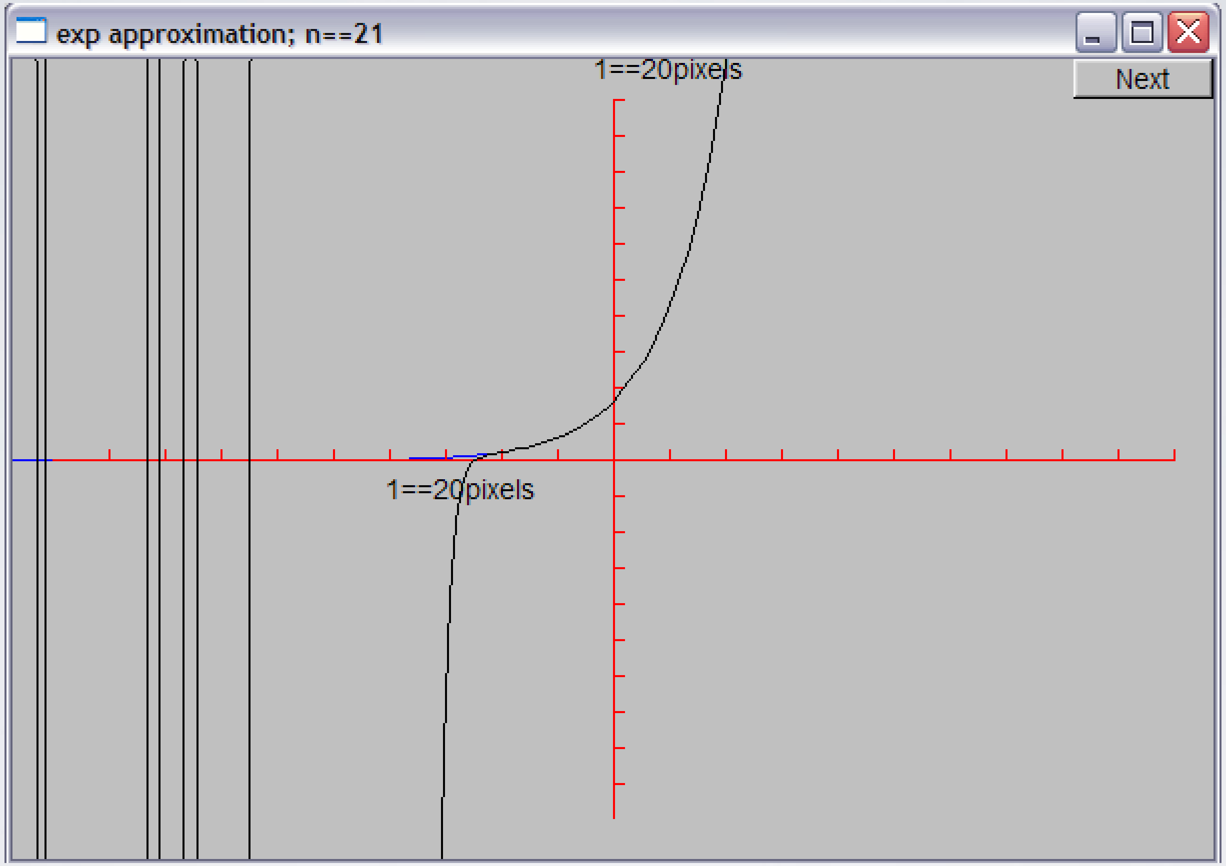
Demo n = 19



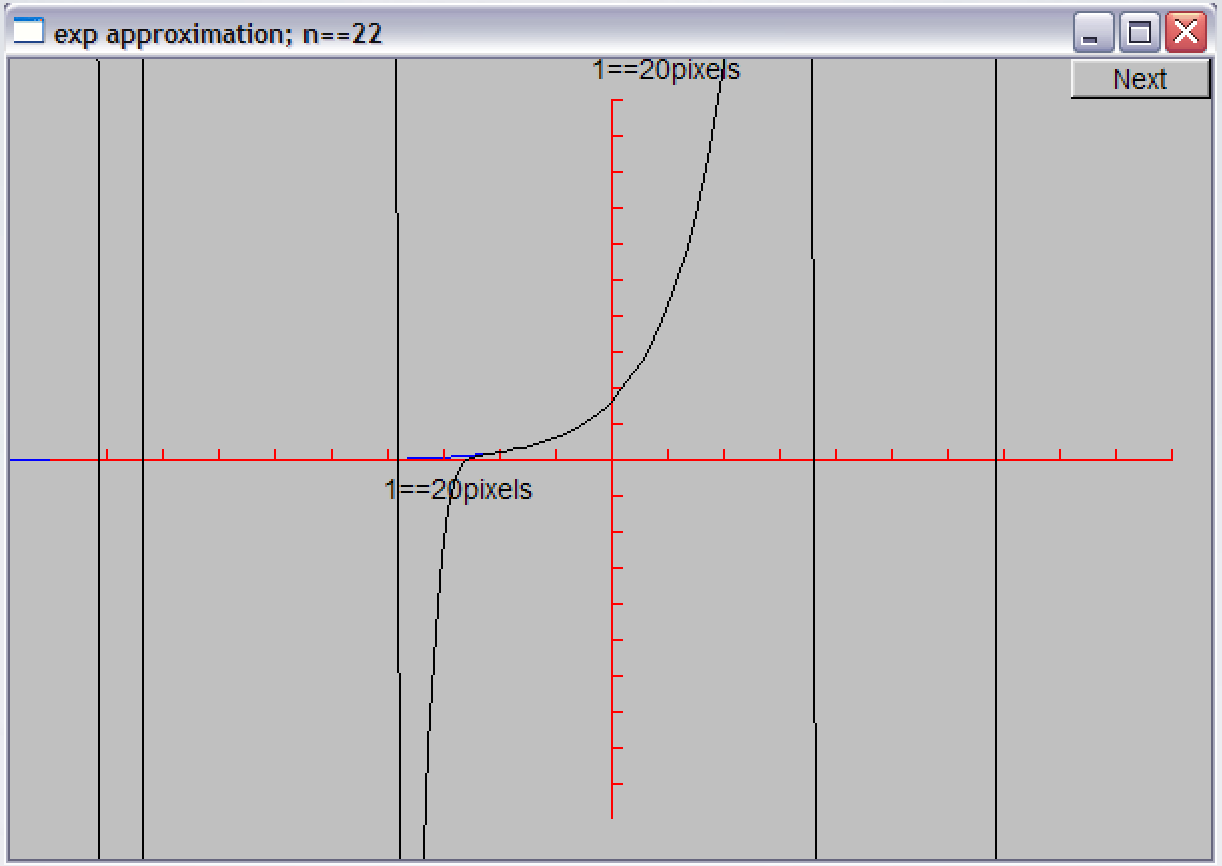
Demo n = 20



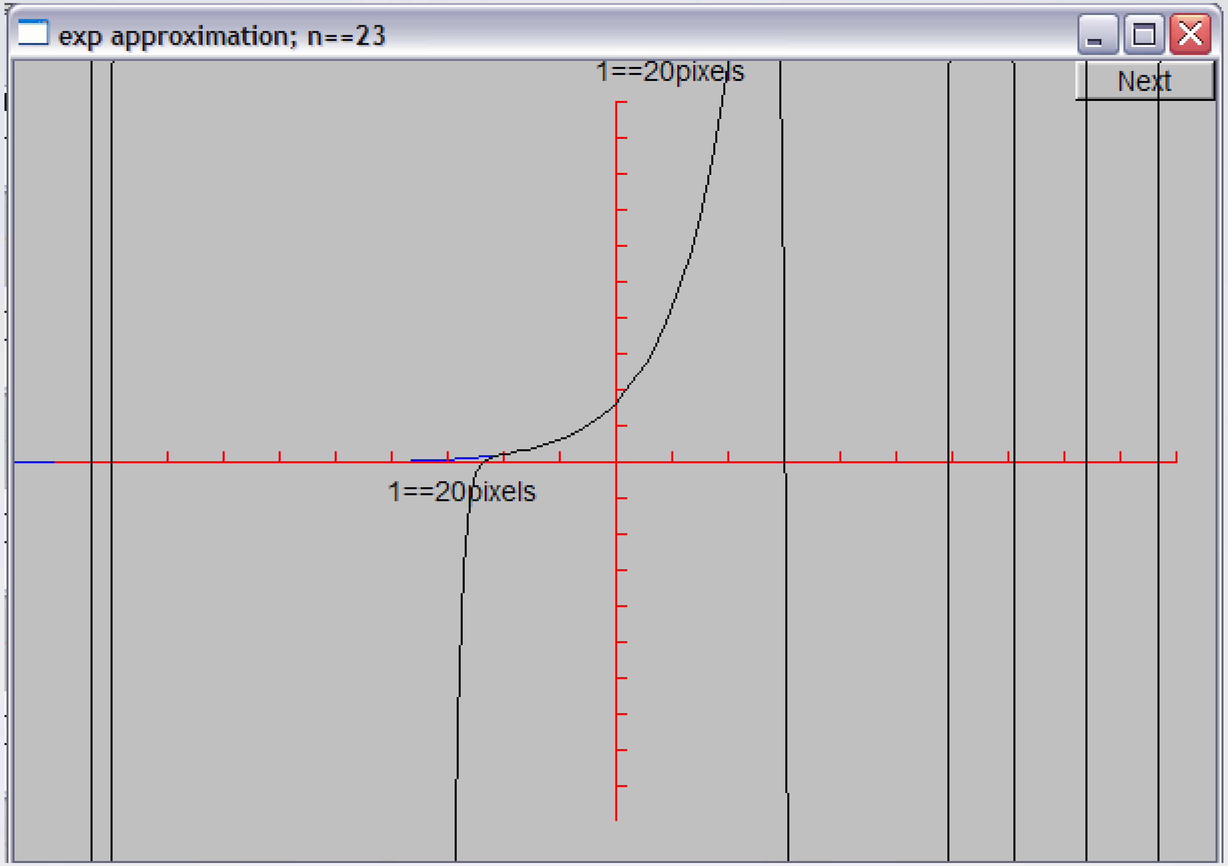
Demo n = 21



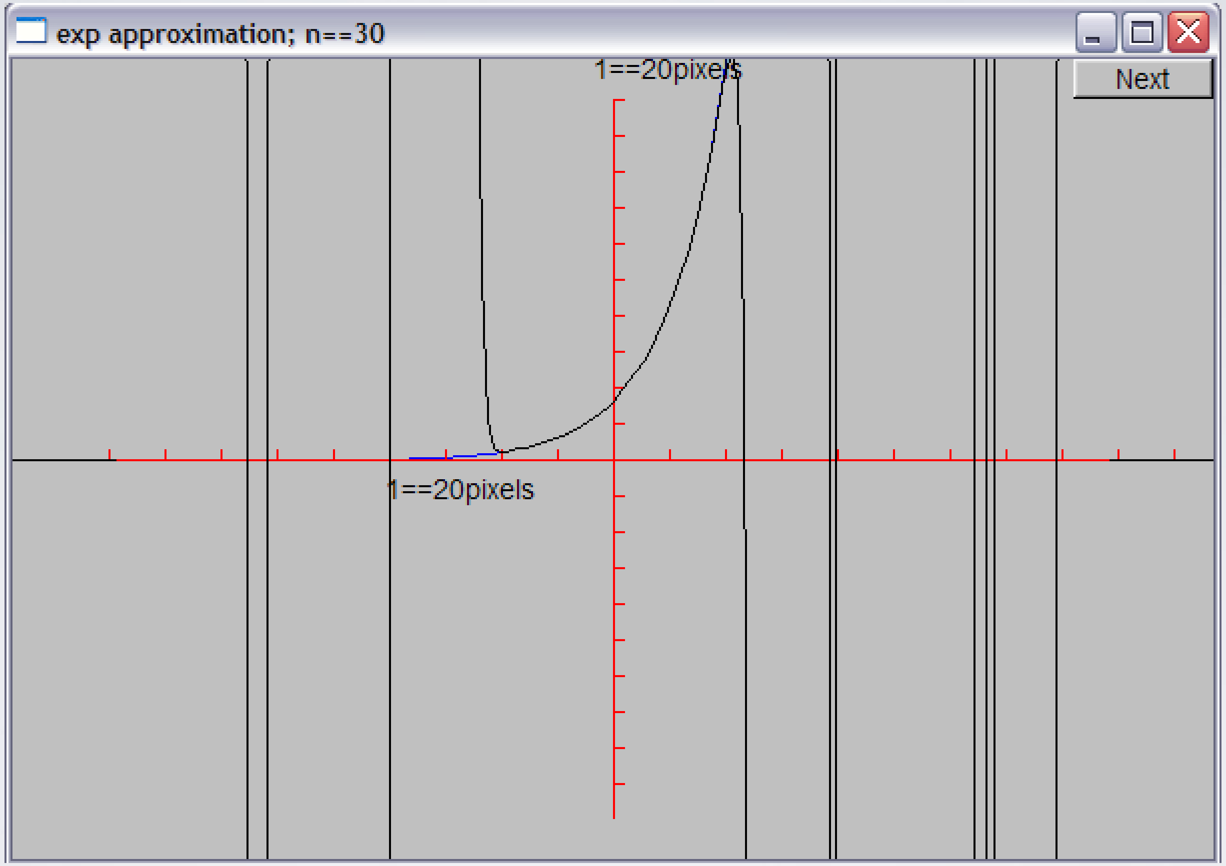
Demo n = 22



Demo n = 23



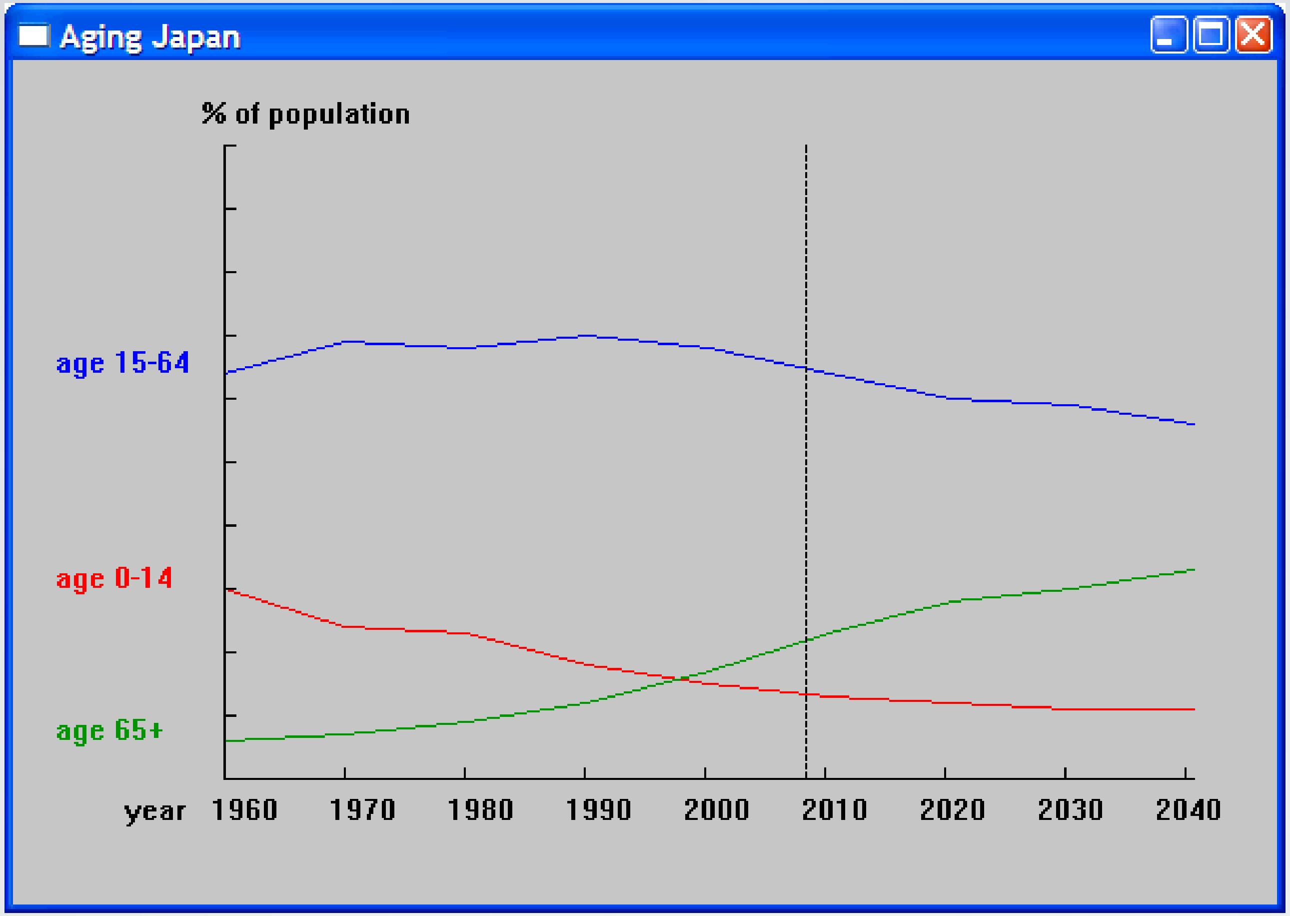
Demo n = 30



Why did the graph "go wild"?

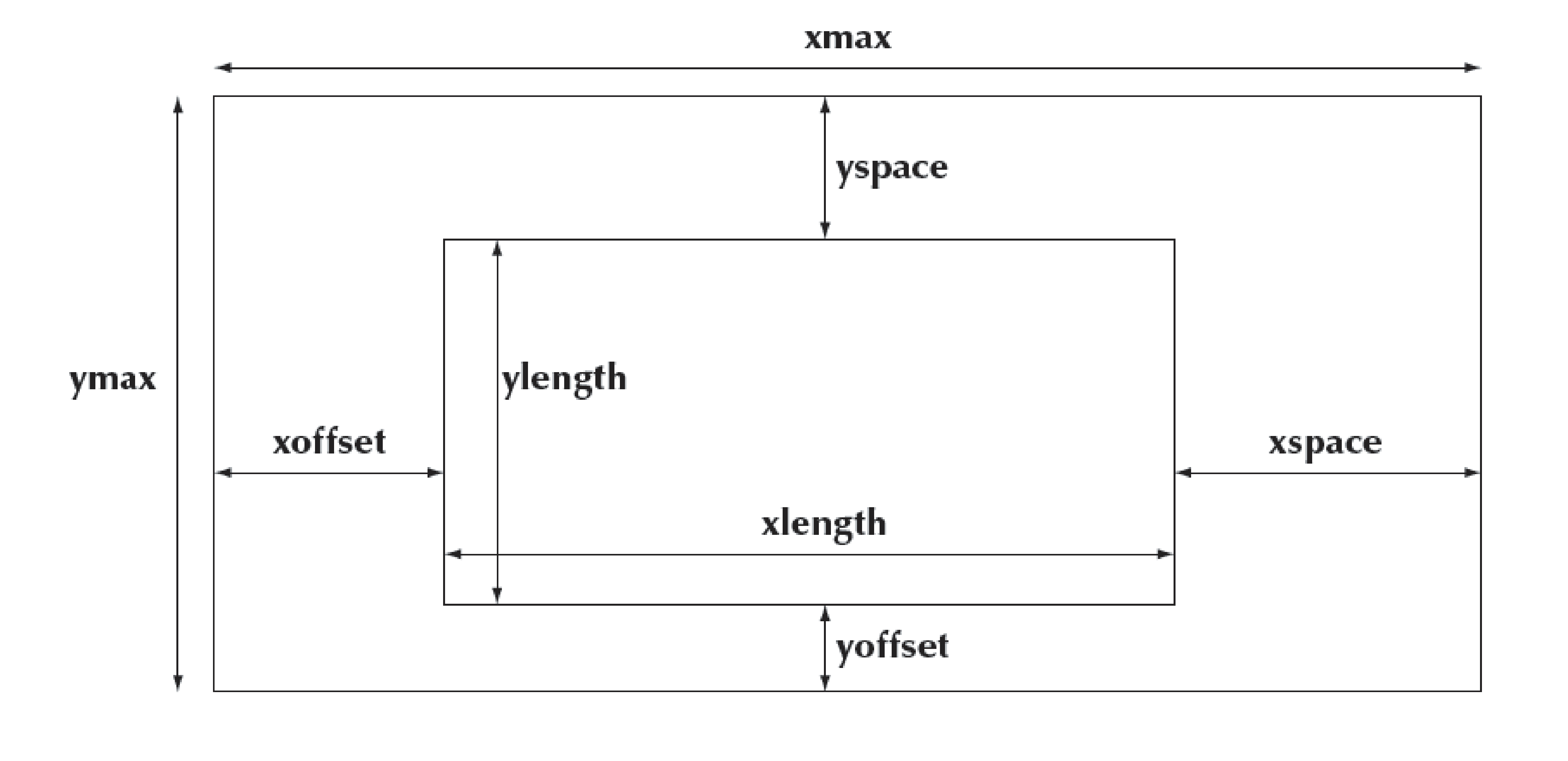
- Floating-point numbers are an approximations of real numbers
  - Just approximations
  - Real numbers can be arbitrarily large and arbitrarily small
    - Floating-point numbers are of a fixed size
  - Sometimes the approximation is not good enough for what you do
  - Here, small inaccuracies (rounding errors) built up into huge errors
- Always
  - be suspicious about calculations
  - check your results
  - hope that your errors are obvious
    - You want your code to break early – before anyone else gets to use it

Graphing data



- Often, what we want to graph is data, not a well-defined mathematical function
  - Here, we used three **Open_polyline**s

Graphing data



- Carefully design your screen layout

Code for Axis

```
struct Axis : Shape {
    enum Orientation { x, y, z };
    Axis(Orientation d, Point xy, int length,
            int number_of_notches=0,        // default: no notches
            string label = ""               // default : no label
            );

    void draw_lines() const;
    void move(int dx, int dy);

    void set_color(Color);         // in case we want to change the color of all parts at once

    // line stored in Shape
    // orientation not stored (can be deduced from line)
    Text label;
    Lines notches;
};
```

```
Axis::Axis(Orientation d, Point xy, int length, int n, string lab)
    :label(Point(0,0),lab)
{
    if (length<0) error("bad axis length");
    switch (d){
    case Axis::x:
    {       Shape::add(xy);                                 // axis line begin
            Shape::add(Point(xy.x+length,xy.y));            // axis line end
            if (1<n) {
                    int dist = length/n;
                    int x = xy.x+dist;
                    for (int i = 0; i<n; ++i) {
                            notches.add(Point(x,xy.y),Point(x,xy.y-5));
                            x += dist;
                    }
            }
            label.move(length/3,xy.y+20);      // put label under the line
            break;
    }
    // …
}
```

Axis implementation

```
void Axis::draw_lines() const
{
    Shape::draw_lines();   // the line
    notches.draw_lines();  // the notches may have a different color from the line
    label.draw();          // the label may have a different color from the line
}

void Axis::move(int dx, int dy)
{
    Shape::move(dx,dy);   // the line
    notches.move(dx,dy);
    label.move(dx,dy);
}

void Axis::set_color(Color c)
{
    // … the obvious three lines …
}
```

Next Lecture

- Graphical user interfaces
- Windows and Widgets
- Buttons and dialog boxes

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Bjarne Stroustrup

Programming -- Principles and Practice Using C++

http://www.stroustrup.com/Programming/

Thank you!

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