

ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ

Συστήματα Διαχείρισης Βάσεων Δεδομένων

Διάλεξη 3η: Access methods: Hash Indexes

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ACCESS METHODS: HASH INDEXES





Hash-based Indexes

Hash Tables

- frequently used as main-memory data structures
- idea: use a hash function h taking as input a (hash) key and returning an integer in the range [0,..,B-1]; B is the number of available buckets
- organization: a bucket array indexed [0,..,B-1] stores the headers of B linked lists; if a record has a search key K, then it will be stored in the list for the bucket numbered h(K)
- Hash Tables in Secondary Storage vs main-memory:
 - Bucket is a unit of storage containing one or more records, typically a disk block
 - Records (or pointers) are placed in the block returned by the hash function
 - Chains of overflow blocks may be added to buckets to accommodate more records
 - Bucket array (directory) consists of block addresses rather than pointers to lists



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Hash Tables in Secondary Storage



Hash file has relative bucket numbers 0 through B-1
Map logical bucket numbers to physical disk block addresses



How do we Choose a Hashing Function ?

- The hash function used should be such that the resulting integer appears to be a random function of the search key
 - thus, buckets will have more or less equal numbers of records and the average time to locate a record will be improved
 - but still one bucket may receive more records than another because some values are more "popular" than others (data skewing)
- Typical hash functions perform computation on the internal binary representation of the search-key
- A function commonly used to hash on integer-valued keys is K mod B, which yields an integer in the range 0 to B-1
 - B is often chosen to be a prime number
 - in certain cases it is convenient to use a power of 2
- For character string keys, characters may be treated as integers (integers are summed up, divided by B, and the remainder of the division is taken as the result of the hash function)



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Choice of Good Hash Functions

- Desired property: expected number of record keys/bucket is the same for all buckets (uniform distribution) and is relatively small; ideally all buckets consist of only one page
 - B must be large enough to minimize the occurrence of overflow chains
 - B must not be so large that bucket occupancy is small and too much space is wasted
- The worst hash function maps all search-key values to the same bucket
 - this makes access time proportional to the number of search-key values in the file

Potentially good hash functions: division or multiplication h(K) = (a*K + a') mod B, h(K) = [fractional-part-of (K * \$\overline\$)] * B, \$\overline\$ golden ratio (0.6180339887 = (sqrt(5)-1)/2)

- advantage: B (size of hash table) need not be a prime number but must be irrational
- Read Knuth Vol. 3 for more on good hashing functions









Inserting into Static Hash Tables

- •To insert record with key K:
 - compute h(K)
 - if bucket h(K) has space, insert the record in the block
 - if not, insert it into one of the overflow blocks associated with the bucket
 - if none of them has space, add a new overflow block and store the record there





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Deleting from Static Hash Tables

- If record with search key K is to be deleted:
 - search bucket h(K) for records with value K
 - delete record(s) found
 - optionally consolidate blocks on a chain after deletion
 - not always a good idea to merge blocks of a chain: repeated insertions / deletions may cause blocks to be created / destroyed at each step





Applications of Hash Tables

- Hashing can be used for both primary and secondary indexes
- For a clustered primary index, no separate index structure is required as the hash function dictates the physical organization of the file
 - The examples so far have been clustered primary indexes
- For a secondary index, the hash function computes a location in a secondary index file
 - This location contains a bucket of physical addresses of records that hash to this logical location





How Full should Buckets be?

Space utilization:

- utilization = # keys used in a bucket
 total # keys that fit in a block
- If too small: wasting space
- If too big, overflows significant

• A good space utilization:

- depends on how good the hash function is
- Rule of thumb: usually between 50% and 80%
 - If U < 50%, space is underutilized
 - if U > 80%, overflows become significant



Efficiency of Static Hash Tables

- Searching for a key v:
 - Evaluate h(v)
 - 2 Fetch bucket at h(v)
 - Search bucket
- In an ideal case, we have enough buckets so that each bucket consists of a single block
 - Lookup: one disk I/O
 - Insert / Delete: two disk I/O's
- In such a case, static hash tables (fixed B) perform better than simple dense or sparse indexes, or even B+-trees
- Dynamically growing files produce overflow chains, which negate the efficiency of the hashing algorithm
 - many blocks per chain may need to be searched
 - hence, #blocks/bucket should be as low as possible



Static Hash Tables





- Bucket (primary page) becomes full
 - Why not re-organize the file by doubling # of buckets?
 - Reading and writing all pages is expensive!
- Idea: the hash file size can grow and shrink "on the fly" in response to the size of the data
 - B is allowed to vary
 - Trick lies in how hash function is adjusted!
- Two types of dynamic hashing:
 - Extensible: uses a directory that grows or shrinks depending on the data distribution
 - No overflow buckets
 - Linear: No directory
 - Splits buckets in linear order, uses overflow buckets



First Idea

•Use a family of hash functions based on **h**:

 $h_k(key) = h(key) \mod 2^k$

 The range of hash function has to be extended to accommodate additional buckets (use a sequence of k bits of h(key) for some large k, e.g., 32)

At any given time a unique hash, h_k, is used depending on the number of times buckets h(K) have been split

- bucket addresses use fewer bits (say i bits from the beginning of the sequence); hence, directory will have 2ⁱ entries
- each bucket stores j, indicating the number of bits used for placing the records in this block (j ≤ i)

Use i of k bits output by hash function





Second Idea:

- Use directory of pointers to bucket blocks, i.e., introduce a level of indirection
- The directory's size is a power of 2 and may vary;
 - double # of buckets by doubling the directory
 - splitting just the bucket that overflowed!
- Directory much smaller than file, so doubling it is much cheaper
 - Only one page of data entries is split
- Certain buckets may share a block for simplicity, "0101" represents a (if the total number of records of the buckets fit in a block)
 For simplicity, "0101" represents a record whose key is hashed to "0101" Alternative 2

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Use directory of pointers





Global depth of directory (i)

- First i bits of a binary number to tell which bucket an entry belongs to
- Number of bits needed to express total number of buckets
- E.g., i=2=>4 buckets; i=3=>8 buckets

Increase i by 1 after a directory split

- Distribute entries across a bucket and its split image based on ith bit
- Local depth of bucket (j)
 - Hash values of data entries in bucket agree on the first j bits
 - Increase j by 1 after a bucket split
 - Assign new j to split image
 - If bucket with local depth j = global depth ^u
 is split, then double the directory

Example: k=4,i=1

Holds records whose key hashes to a sequence beginning with 0 0001**Global Depth** i=1 ()**Buckets** 1001 1100Directory Holds records whose key hashes to a sequence beginning with 1 Local Depth: Number of bits used to determine membership of records in block 19







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Insertion into Extensible Hash Tables

- To insert a record with key K:
 - compute h(K); take the first i bits of the sequence returned and follow the pointer in the directory indexed by these i bits (global depth), leading to block b
 - if there is room, place record in b
 - if not, then, let j be the local depth stored in the block header b:
 - 1. If $\mathbf{j} < \mathbf{i}$ (local < global), then:
 - a) Split **b** in two
 - b) Distribute records in b to the two blocks based on the value of the (j+1)-st bit: those with 0 stay in b and those with 1 go to the new block
 - c) Update header of each block with the split image j+1
 - d) Adjust pointers in directory so that entries point to the correct block depending on their (j+1)-st bit



Insertion into Extensible Hash Tables

- It may so happen, that examining the j-th bit may send all records to one block
 - In this case, the process is repeated with the next higher value of the split image j
- 2. If $\mathbf{j} = \mathbf{i}$ (local = global), then:
 - a) increment i by 1; double the size of the directory so it now has 2ⁱ⁺¹ entries
 - b) let w be a sequence of i bits indexing the entries of the previous directory
 - Then w_0 and w_1 each point to the same block that w pointed to (entries share the block but the block hasn't changed)
 - c) As in case 1, split b (since i>j)



Example: insert record with value hashing to 1010 in the table





Insertion into Extensible Hash Tables

Example: insert records with values hashing to 0000 and 0111



- first block overflows
- split block
- j < i; j++





Insertion into Extensible Hash Tables









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Deletion from Extensible Hash Tables

- To delete record with key K:
 - remove data entry from bucket
 - if bucket is empty
 - merge bucket with its split image
 - decrease local depth
 - If each directory element points to the same bucket as its split image
 - Halve directory
 - Reduce global depth



Analysis of Extensible Hash Tables

 With uniform distributed addresses, all the buckets tend to fill up at the same time => split at the same time (Periodic and fluctuating)

As buffer fills up to 90%

After a concentrated series of splits drops to under 50%

Space utilization of the bucket

- R (# of records), b (block size), B (# of Blocks)
- •Utilization = R / b*B
- Average utilization $\sim 1n$ 2 (natural logarithm) = 0.69 ==> 69%
 - recall normal B+-tree: 67%, B+-tree with redistribution: 85 %
- Space utilization for the directory
 - How large a directory should we expect to have, given an expected number of keys?

• estimated directory size = $3.92/b*R^{(1+1/b)}$ Flajolet (1983)

- Overall Load Factor (LF) of the file below 2 seeks, 75%~80% utilization Litwin (1980)
 - LF: The number of records in the file divided by the number of places for the records in the primary area



A Pathological Case





Efficiency of Extensible Hash Tables

 Example: 100MB file, 100 bytes/record, 4K pages contains 1.000.000 records (as data entries) and 25.000 directory entries

- chances are high that directory will fit in memory
 - With 80% utilization the directory size will be
 - 1.2 * 25.000 * 100 = 3.000.000 = 3M
- Pros:

equality search can be answered with only one I/O to locate the record

- if the size of the bucket directory is small enough to be kept in main memory no disk I/O is required to look-up the directory
- Cons:
 - when the size of the bucket directory is doubled (if i is already large), it may no longer fit in main memory
 - if the number of records per block is small, block splitting may occur earlier than actually required
- Can we do better? (smoother growth of bucket directories)
 - In Linear Hashing, buckets are split from left to right, regardless of which one overflowed (simple, but it works!!)



Directory Doubling: Least vs. Most Significant Bits

Global depth of directory

Max # of bits needed to tell which bucket an entry belongs to

Local depth of a bucket

of bits used to determine if an entry belongs to this bucket

- Use of least (vs most) significant bits enables efficient directory doubling via copying!
 - doubled by copying it over and `fixing' pointer to split image page









Linear Hashing

- During round number Level, only hash functions h_{Level} and h_{Level + 1} are used, so at any given point in a round, we have:
 - buckets that have been split (0 to Next-1)
 - buckets that are yet to be split (Next to B_{level}-1)
 - buckets created by splits in this round (B_{Level} to 2B_{Level}-1 < B_{level+1})



Linear Hashing

•In the middle of a round







Linear Hashing: Bucket Splitting

- Allocate a new bucket, append it to the hash table (its position will be 2^{Level} B + Next)
- Re-distribute the entries in bucket Next by rehashing them via h_{Level+1} (some entries remain in bucket Next, some go to bucket B*2^{Level}+Next)





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Linear Hashing: Rehashing

Hashing via h_{Level} has to take care of Next's position:
 h_{Level}(key) = { < Next: we hit an already split bucket, rehash >= Next: we hit a yet unsplit bucket, bucket found





Linear Hashing: Split Rounds

A bucket split increments Next by 1 to mark the next bucket to be split

- How would you propose to handle the situation if Next is incremented beyond the last current hash table position, i.e., Next> 2^{Leve}B- 1?
 - If Next> 2^{Level}B- 1, all buckets in the current hash table are hashed via h_{Level+1} (see previous slides)
 - Linear hashing thus proceeds in a round-robin fashion:
 - If Next> 2^{Leve}B- 1, then
 - increment level by 1,
 - reset next to 0 (start splitting from the hash table top again)
- In general, an overflowing bucket is not split immediately, but—due to round-robin splitting—no later than in the following round



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Insertion into Linear Hash Tables

- To insert a data entry K:
 - •Locate bucket by applying $\mathbf{h}_{Leve1}(\mathbf{K})$ or $\mathbf{h}_{Leve1+1}(\mathbf{K})$
 - If bucket is full
 - Add overflow page and insert data entry
 - (Maybe) Split bucket Next
 - Hash function $h_{Leve_{1+1}}$ redistributes entries between bucket Next and bucket Next + $B_{Leve_{1}}$
 - Increment pointer Next
- Buckets are split round-robin, long overflow chains don't occur
 in general, the spitted bucked is not the bucket which triggered the split!
 each bucket may use overflow pages
- Choose any criterion to `trigger' split
 - insertions filled a primary bucket page beyond c % capacity,
 - or the overflow chain of a bucket grew longer than p pages, or . . .
- We will examine, in the sequel, the first growth method



Example of Linear Hashing

• On split, **h**_{l evel+1} is used to re-distribute entries









Analysis of Linear Hashing

- B: the current number of bucket blocks in use (at round number Level)
 - As the table grows, the initial # of bucket blocks is increased but it may not always be a power of 2
- Level: the # bits of h(k) used to address buckets is [log2B] (low-order bits)
- R: number of records stored in hash table
 - initially 0 and increased as records are added
- Utilization of the hash table is the number of stored records divided by the number of possible storage places = R / B * c (#entries/block)
 - ratio R/B limited so that the average number of records per bucket is a fixed fraction of the number of records that fit in one block
 - •overflow blocks are permitted, but the average number of overflow per bucket will be <= 1</p>

We will use this ratio to determine when to grow or shrink the hash table
 grow when R/B >= 90% and shrink when R/B <= 75%

we will use the average value of 85% as the threshold for growth



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Insertion into Linear Hash Tables: Alternative Algo

- To insert record with key K:
 - Compute h(K)=b and determine the i-bit rightmost (b=a₁a₂...a_i) sequence of h(K) to use as the bucket number
 - If b<B the bucket with number b exists and the record is placed there
 - If B≤b<2ⁱ, then the bucket does not exist and we have to place the record in bucket b-2ⁱ⁻¹ (this is the bucket we would obtain if a₁ changed to 0)
 - If the bucket we add has number 1a₂...a_i, then we split the bucket numbered 0a₂...a_i and divide the records depending on their last i bits
 - If $B>2^i$ then i is incremented by 1 (leading 0 is added)
 - Each time an insertion need to be performed, we examine the ratio R/B and if it is too high, we increase B by 1 (R the # of records in hash table)



Linear Hashing Occupancy Example

Example: B=2, Level=1, assume h produces 4-bit sequences (k=4)



 If R <=1.7B, and block holds 2 records then on the average, occupancy c of a bucket does not exceed 85% (i.e., ≤ 1.7/2) of the block capacity



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Linear Hashing: Second Example

Example: 2 keys/block, R=3, B=2, Level =1 assume h produces 4-bit sequences (k=4)





-> R=4, B=3, Level =2; distribute keys between buckets 00 and 10:



•now R=4 <1.7B (R/B = 1.33) !



R=4, B=3, Level =2; insert 00<u>01</u>:













Overflow block no longer needed !

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Example Continued: How to grow beyond this?





Linear vs. Extensible Hashing

Two schemes are quite similar

- Imagine Linear Hashing (LH) has a directory with elements 0 to B-1
- ◆ First split at bucket 0, add directory element B
 - Imagine entire directory doubled at this point
 - Elements <1,B+1>, <2,B+2>, ... are the same
 - Only create directory element **B**, which differs from 0, now
- Second split at bucket 1, create directory element B+1
- So, directory can double gradually
 - Also, primary bucket pages are created in order
 - If they are allocated in sequence too, then finding bucket i is easy
 - Thus, we actually don't need a directory!
- LH is recommended when main storage is at a premium since it requires no directory
 - Particularly useful in a small computer environment
- EH could be useful if sufficient main memory is available to hold the directory
 - Doubling and halving the directory size is expensive



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Hashing vs. B+ trees

- In a typical DBMS, you will find support for B+ trees as well as hash-based indexing structures
 - Cost of periodic re-organization
 - Relative frequency of insertions and deletions
 - Is it desirable to optimize average access time at the expense of worstcase access time?
- In a B+ tree, to locate a record with key k means to compare k with other keys k' organized in a (tree-shaped) search data structure
- Hash indexes use the bits of k itself (independent of all other stored records and their keys) to find the location of the associated record
- Hash indexes: best for equality searches, cannot support range searches (also known as scatter storage)
 - True, but they can be used to answer range queries in O(1+Z/B) I/Os, where Z is the number of results (Alstrup et all 2001; Brodal et all 2004)
- Although hash indexes supports multiple attribute keys, cannot support partial key search



References

- Based on slides from:
 - R. Ramakrishnan and J. Gehrke
 - H. Garcia Molina
 - ◆J. Hellerstein
 - L. Mong Li
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 - R. Lawrence
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Τέλος Ενότητας







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