ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ

Ψηφιακή Οικονομία

Διάλεξη 10η: Basics of Game Theory part 2

Μαρίνα Μπιτσάκη
Τμήμα Επιστήμης Υπολογιστών

Best Response Curves

- Used to solve for equilibria in games
- A player's best response is the choice that maximizes its pay-off, taking other players' strategies given
- Consider a game of two players A and B
  - $b_A(r)$ best response for A when B's strategy is r
  - $b_B(c)$ best response for B when A's strategy is c
  - A Nash equilibrium is a pair of strategies (c^*, r^*) such that

    $c^* = b_A(r^*)$

    $r^* = b_B(c^*)$

Example

- Player's A strategies: Top with probability p, Bottom with probability $1-p$
- Player's B strategies: Left with probability q, Right with probability $1-q$

		B	
A		Left	Right
	Top	0, 0	0, -1
	Bottom	1, 0	-1, 3

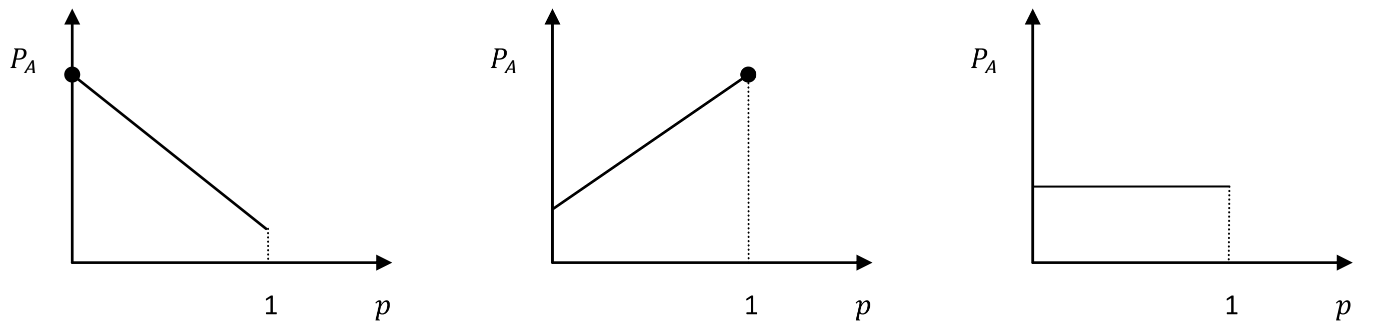
- Profit of A: $\boldsymbol{P_A}(\boldsymbol{p}) = q(1-p) + (1-q)\big(-(1-p)\big) = q - qp + (1-q)(p-1) = (-2q+1)p + 2q - 1$
- Profit of B: $\boldsymbol{P_B}(\boldsymbol{q}) = p\big(-(1-q)\big) + 3(1-p)(1-q) = (4p-3)q - 4p + 3$

Maximization problem for A: $\max_{0\leq p\leq 1}[(-2q+1)p+2q-1]$

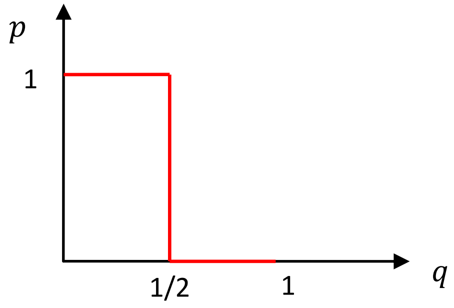
If $(-2q+1<0)\Rightarrow q>\frac{1}{2}$ then maximum is reached at $p=0$

If $(-2q+1>0)\Rightarrow q<\frac{1}{2}$ then maximum is reached at $p=1$

If $(-2q+1=0)\Rightarrow q=\frac{1}{2}$ then maximum is reached at any $0\leq p\leq 1$



Best response curve $p(q)$ for A:

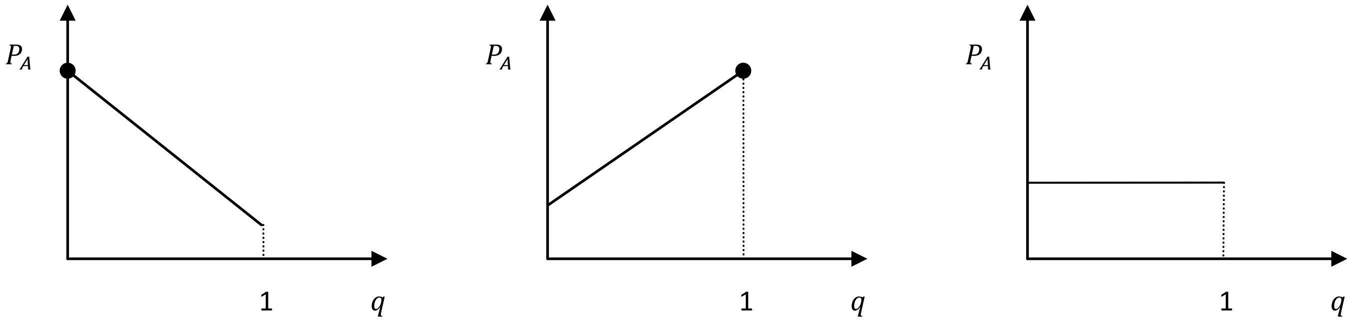


Maximization problem for B: $\max_{0\le q\le 1}[(4p-3)q-4p+3]$

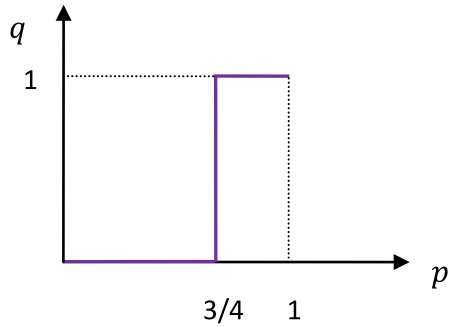
If $(4p-3<0) \Rightarrow p<3/4$ then maximum is reached at q$=0$

If $(4p-3>0) \Rightarrow p>3/4$ then maximum is reached at $q=1$

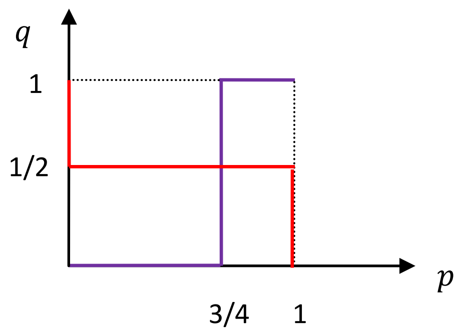
If $(4p-3=0) \Rightarrow p=3/4$ then maximum is reached at any $0\le q\le 1$



Best response curve $q(p)$ for B:



Both best responses cross at $(p, q) = (\frac{3}{4}, \frac{1}{2})$, which is the unique Nash equilibrium



Example

	B		
A		Left	Right
	Top	2, 1	0, 0
	Bottom	0, 0	1, 2

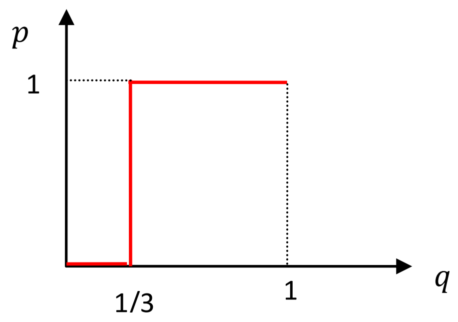
$P_A = q(2p) + (1-q)(1-p) = 3pq - p - q + 1$

Let p change by Δp then payoff changes by $\Delta P_A = 3q\Delta p - \Delta p = (3q-1)\Delta p$

A wants to increase p $(\Delta p > 0)$ whenever $3q - 1 > 0 \Leftrightarrow q > 1/3$ $(\max \Delta P_A \rightarrow \max \Delta p \longrightarrow p = 1)$

A wants to decrease p $(\Delta < 0)$ whenever $3q - < 0 \Leftrightarrow q < 1/3$ $(\max \Delta P_A \rightarrow \max \Delta p \longrightarrow p = 0)$

A is happy with any value of $0 \leq p \leq 1$ when $q = 1/3$



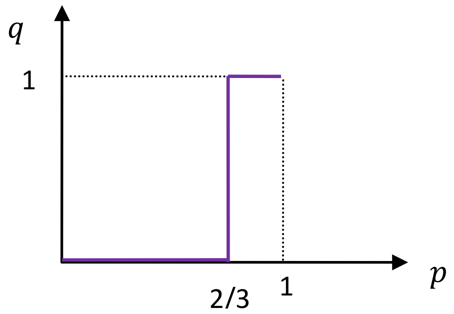
$P_B = pq + 2(1-p)(1-q) = 3pq - 2p - 2q + 2$

Let q change by Δq then payoff changes by $\Delta P_B = (3p-2)\Delta q$

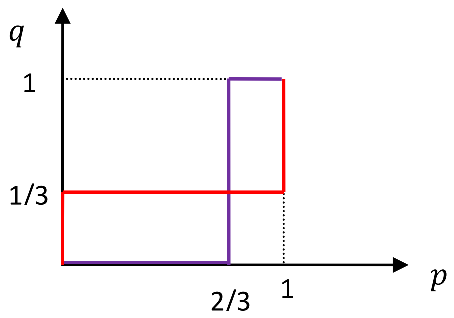
If $p > 2/3$ B wants to increase q at $q = 1$

If $p < 2/3$ B wants to decrease q at $q = 0$

If $p = 2/3$ B chooses any value of $0 \leq q \leq 1$ as best response



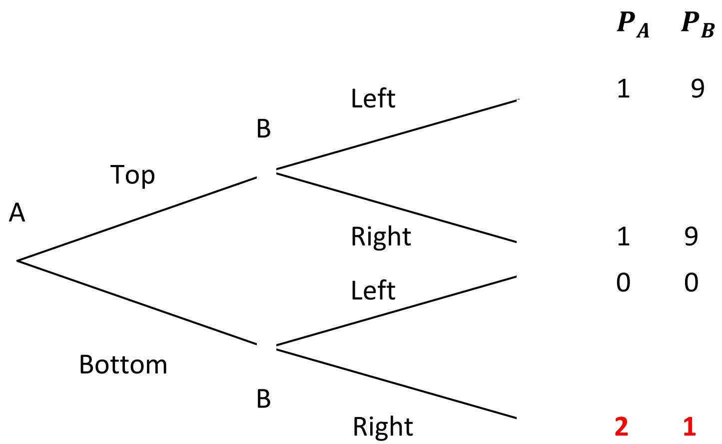
Nash equilibria are: (0,0), (1,1), (2/3, 1/3)



Sequential Games

- Simultaneous game: Nash equilibria → (Top, Left), (Bottom, Right)
- <u>Sequential game</u>: A chooses first, B observes A and chooses next
  - (Top, Left) is not reasonable in the sequential game
- Extensive form of the game

		B	
		Left	Right
A	Top	1, 9	1, 9
	Bottom	0, 0	2, 1



Repeated Games

- Each player has the opportunity to establish a reputation for cooperation
- If a player does not cooperate the opponent can punish for bad behavior
- When a game is repeated a <u>fixed</u> number of times, players do not cooperate
- When a game is repeated an <u>indefinite</u> number of times then the strategy with the highest overall pay-off is ‘tit for tat’
  - ‘tit for tat’: do whatever the other player did in the last row

Example

	B		
A		Defect	Cooperate
	Defect	**-3, -3**	0, -6
	Cooperate	-6, 0	-1, -1

Defect is dominant strategy for A and B in a one-shot game

i. Repeated number of times $n = 10$

At round 10: Both players defect (choose their dominant strategies) – same outcome as if only 1 round

At round 9: If player A cooperates then B will defect knowing round's 10 outcome – so both defect

There is no way to enforce cooperation – **same outcome as in a one-shot game**

ii. Indefinite number of times

Best strategy for A (tit for tat):

Round 1: cooperate

Round 2: if B defected at round 1 defect otherwise cooperate

…

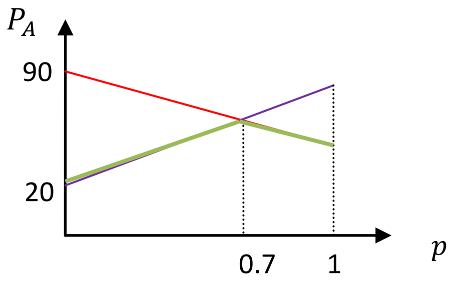
Games of competition

Zero –sum games: payoff to one player is equal to the losses of the other

Example (penalty point in soccer)

	B			
A		Defend Left	Defend Right	$\boldsymbol{P_A}$
	Kick Left	50, -50	80, -80	$\mathbf{50q + (1-q)80}$
	Kick Right	90, -90	20, -20	$\mathbf{90q + (1-q)20}$
	$\boldsymbol{P_A}$	$\mathbf{50p + 90(1-p)}$	$\mathbf{80p + 20(1-p)}$	

Payoff maximization for A



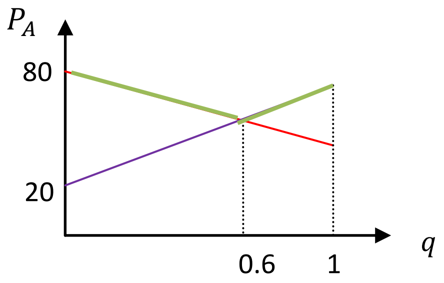
Red line: A’s payoff if B chooses “Defend Left”

Purple line: A’s payoff if B chooses “Defend Right”

Green line: for each p the best payoff of A (P_A) since B will try to minimize A’s payoff

Maximum payoff is reached at $p = 0.7$

Payoff maximization for B



Red line: A’s payoff if A chooses “Kick Left”

Purple line: A’s payoff if A chooses “Kick Right”

Green line: for each q the best payoff of A (P_A) (A tries to maximize B’s losses)

Maximum payoff for B is reached at $q = 0.6$ (the point at which A’s maximum payoff is minimized)

Coordination Games

Pay-offs are highest when players coordinate their strategies

Example: battle of the sexes

- Nash Equilibrium: (art, art), (action, action), (2/3, 2/3) mixed strategy
- How to coordinate: one player moving first

	Girl		
Boy		Action	Art
	Action	2, 1	0, 0
	Art	0, 0	1, 2

Τέλος Ενότητας

Ευρωπαϊκή Ένωση
Ευρωπαϊκό Κοινωνικό Ταμείο
ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ & ΘΡΗΣΚΕΥΜΑΤΩΝ, ΠΟΛΙΤΙΣΜΟΥ & ΑΘΛΗΤΙΣΜΟΥ
ΕΙΔΙΚΗ ΥΠΗΡΕΣΙΑ ΔΙΑΧΕΙΡΙΣΗΣ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης
ΕΣΠΑ 2007-2013
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ

Χρηματοδότηση

•Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στα πλαίσια του εκπαιδευτικού έργου του διδάσκοντα.

•Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Κρήτης**» έχει χρηματοδοτήσει μόνο τη αναδιαμόρφωση του εκπαιδευτικού υλικού.

•Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Ευρωπαϊκή Ένωση
Ευρωπαϊκό Κοινωνικό Ταμείο
ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΘΡΗΣΚΕΥΜΑΤΩΝ
ΕΙΔΙΚΗ ΥΠΗΡΕΣΙΑ ΔΙΑΧΕΙΡΙΣΗΣ
ΕΣΠΑ 2007-2013
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Σημειώματα

Σημείωμα αδειοδότησης

Σημείωμα Αναφοράς

Copyright Πανεπιστήμιο Κρήτης, Μαρίνα Μπιτσάκη. «**Ψηφιακή Οικονομία. Διάλεξη 10η: Basics of Game Theory part 2**». Έκδοση: 1.0. Ηράκλειο/Ρέθυμνο 2015. Διαθέσιμο από τη δικτυακή διεύθυνση: https://elearn.uoc.gr/course/view.php?id=420/