Introduction to Cognitive radios
Part one

HY 539

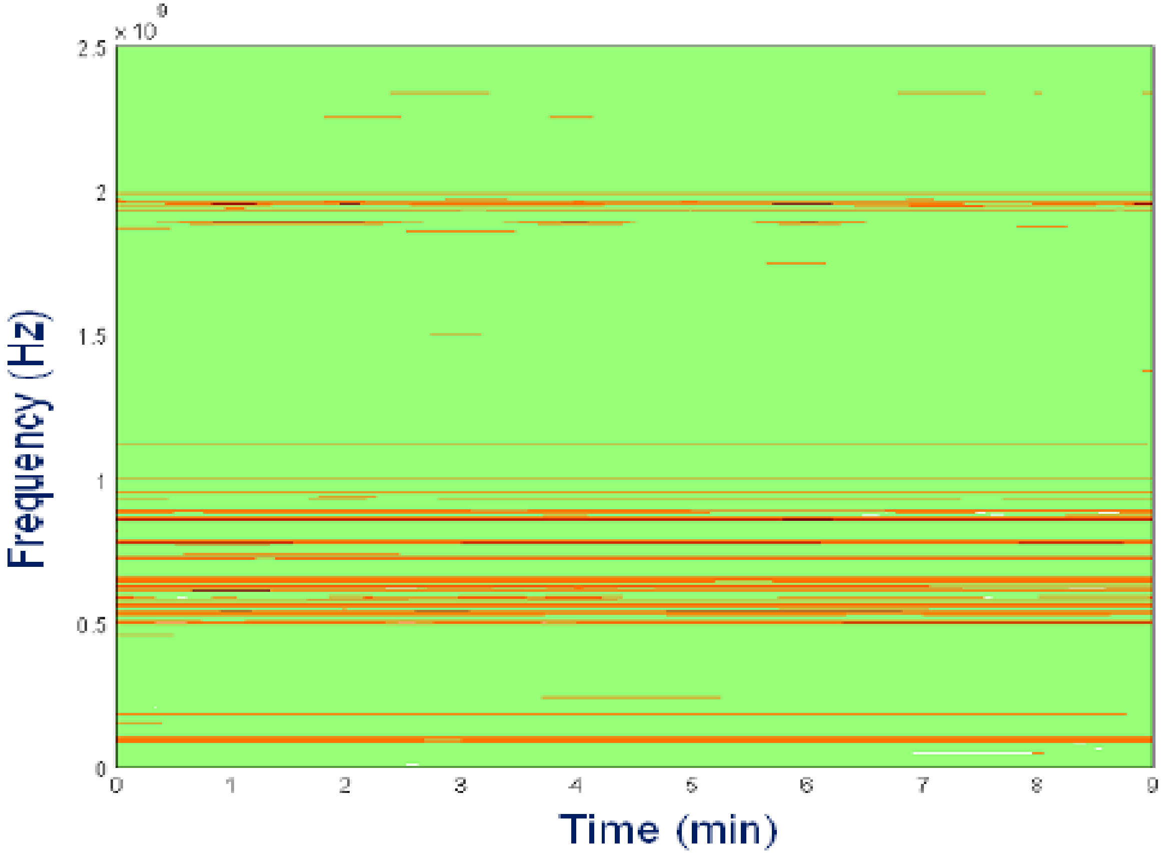
Presented by: George Fortetsanakis

Increased user demand

- The ISM band is a host of many different wireless technologies.
  - WiFi
  - Bluetooth
  - Wimax

- The number of devices that function at the ISM band is constantly growing.
  - Interference between these devices is growing as well.
  - This means degradation of performance.

Underutilization of licensed spectrum



- Licensed portions of the spectrum are underutilized.
  - According to FCC, only 5% of the spectrum from 30 MHz to 30 GHz is used in the US.

Cognitive radios

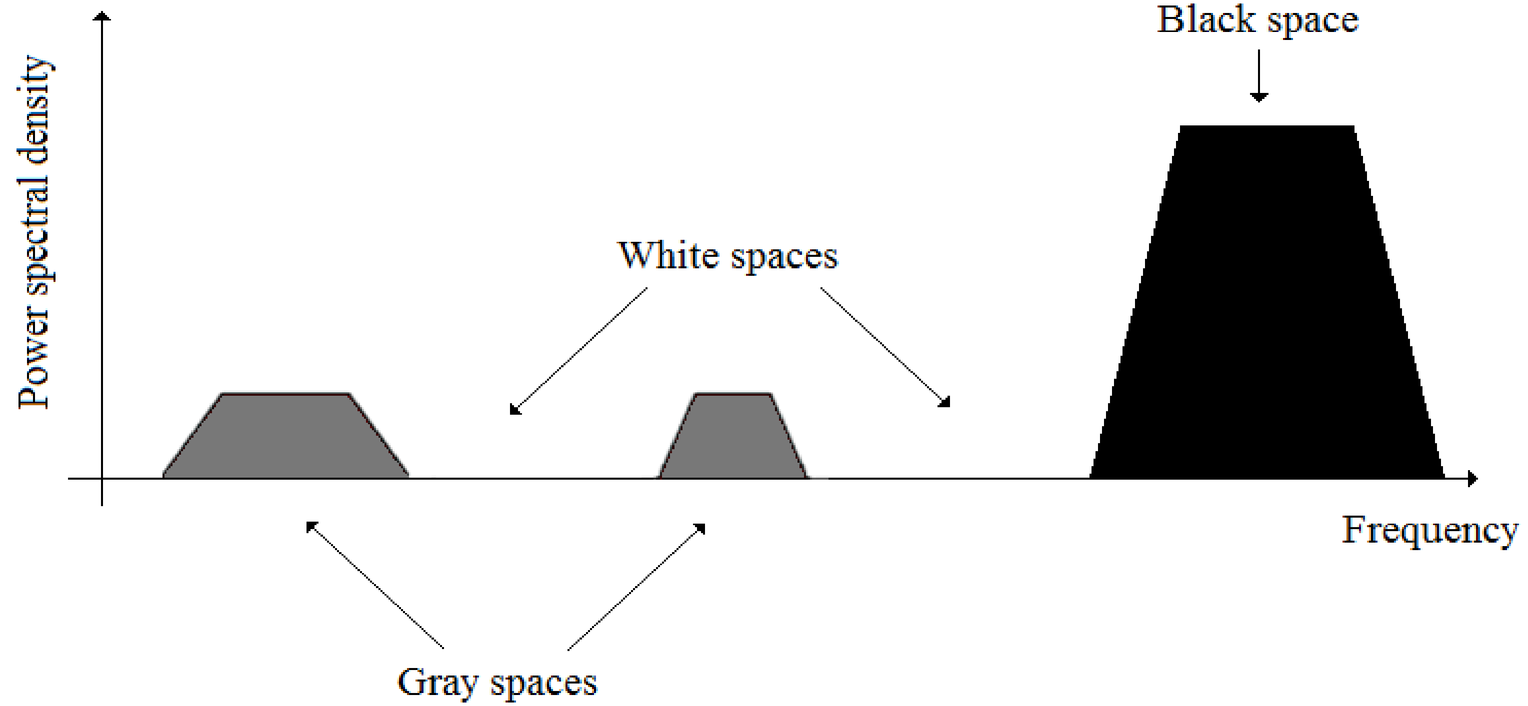
- Intelligent devices that can coexist with licensed users without affecting their quality of service.
  - Licensed users have higher priority and are called **primary users**.
  - Cognitive radios access the spectrum in an opportunistic way and are called **secondary users**.

- Networks of cognitive radios could function at licensed portions of the spectrum.
  - Demand to access the ISM bands could be reduced.

Restrictions to secondary users

- Licensed portions of the spectrum consists of frequency bands that belong to one of the following categories:
  - **White spaces:** Primary users are absent. These bands can be utilized without any restriction.
  - **Gray spaces:** Primary users are present. Interference power at primary receivers should not exceed a certain threshold called interference temperature limit.
  - **Black spaces:** Primary user's power is very high. Secondary users should use an interference cancellation technique in order to communicate.

Example



- Secondary users can identify white, gray and black spaces and adapt according to the corresponding restrictions.

Coexistence of secondary users

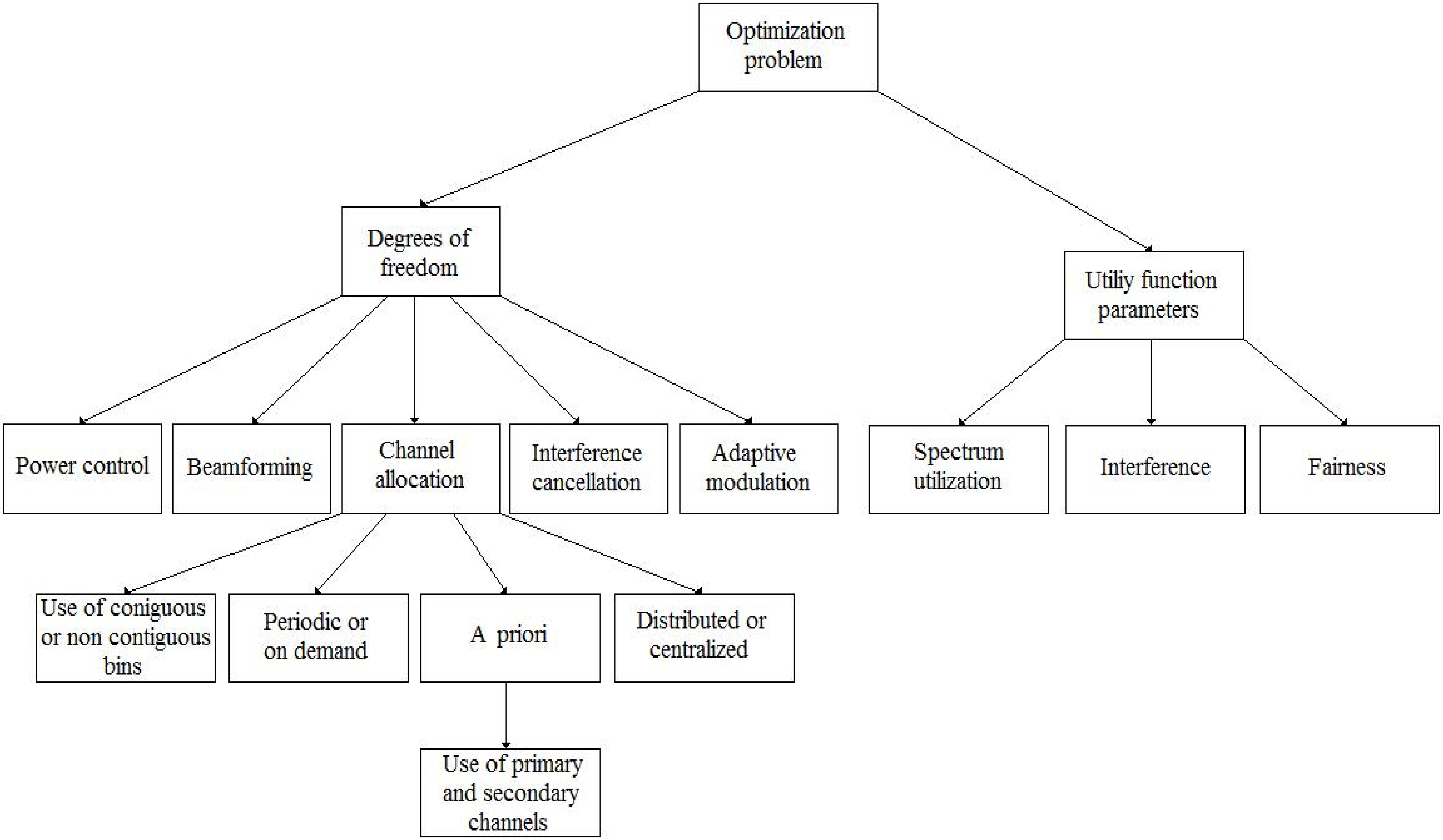
- Usually, in cognitive radio networks, a large number of secondary users compete to access the spectrum.
- A **protocol** should define the behavior of all these users such that the network's performance is maximized.
- Performance metrics:
  - Spectrum utilization
  - Fairness
  - Interference to primary users.

Performance optimization

- Proposed protocols in the literature define an optimization problem.
  - The utility function depends on the performance metrics.

- Parameters of the problem are chosen from the following set:
  - Channel allocation
  - Adaptive modulation
  - Interference cancellation
  - Power control
  - Beamforming

Definition of the problem



Optimization problem

Degrees of freedom

Utiliy function parameters

Power control

Beamforming

Channel allocation

Interference cancellation

Adaptive modulation

Spectrum utilization

Interference

Fairness

Use of coniguous or non contiguous bins

Periodic or on demand

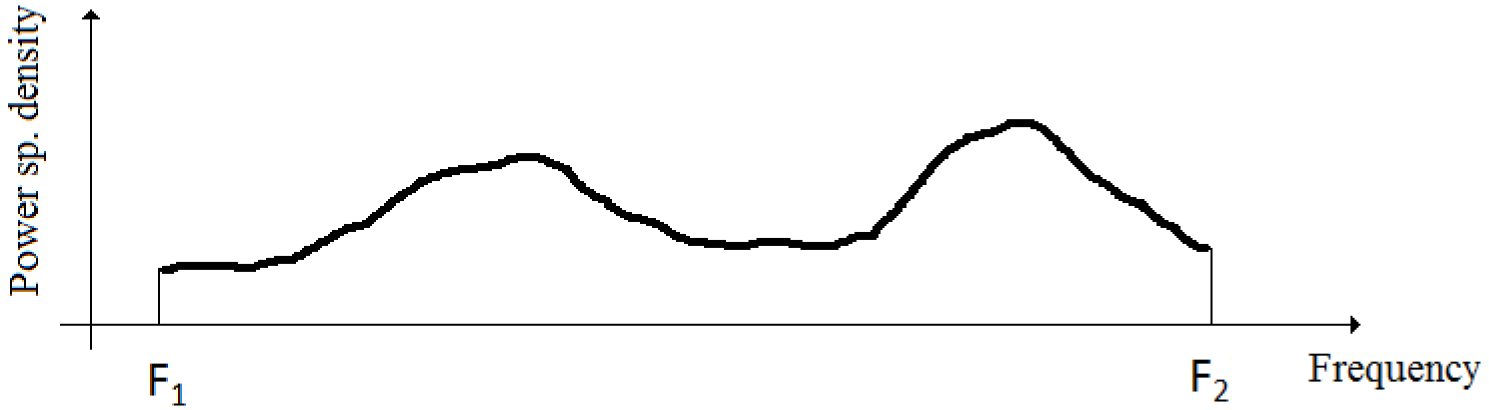
A priori

Distributed or centralized

Use of primary and secondary channels

1. Channel allocation

- Problem formulation:
  - 2 secondary users compete for access in the band [F_1 F_2].
  - The interference plus noise power as observed by the first user is:

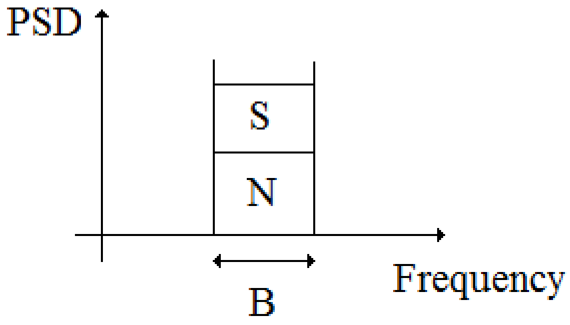


- Question: Which is the best way for this user to distribute its transmission power at the interval [F_1 F_2]?

Channel capacity

- According to Shannon the maximum rate that can be achieved in a channel is:

$$R(S) = B\log_2\left(1+\frac{S}{N}\right)$$

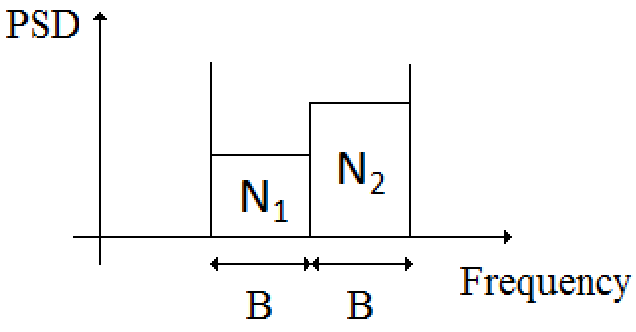


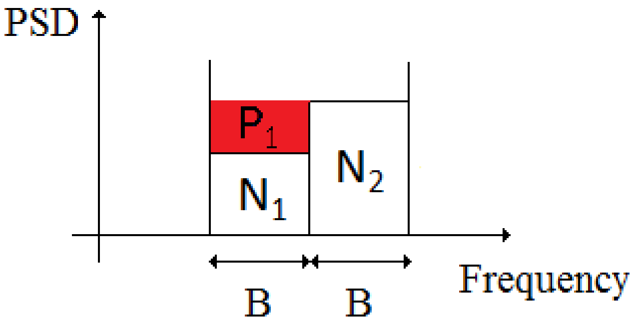
- S: signal power
- N: interference plus noise power
- B: width of the channel

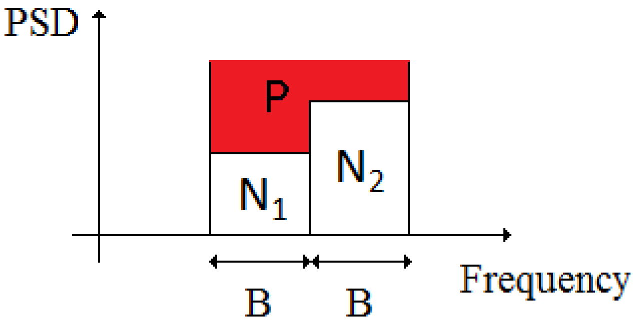
$$\frac{dR(S)}{dS} = \frac{B}{\ln 2}\frac{1}{1+\frac{S}{N}}\frac{1}{N} = \frac{B}{\ln 2}\frac{1}{S+N}$$

- As the power that is introduced to a channel increases, the achievable rate increases more and more slowly.

Energy investment in two channels







$$\frac{B}{\ln 2}\frac{1}{N_1} > \frac{B}{\ln 2}\frac{1}{N_2} \Rightarrow$$

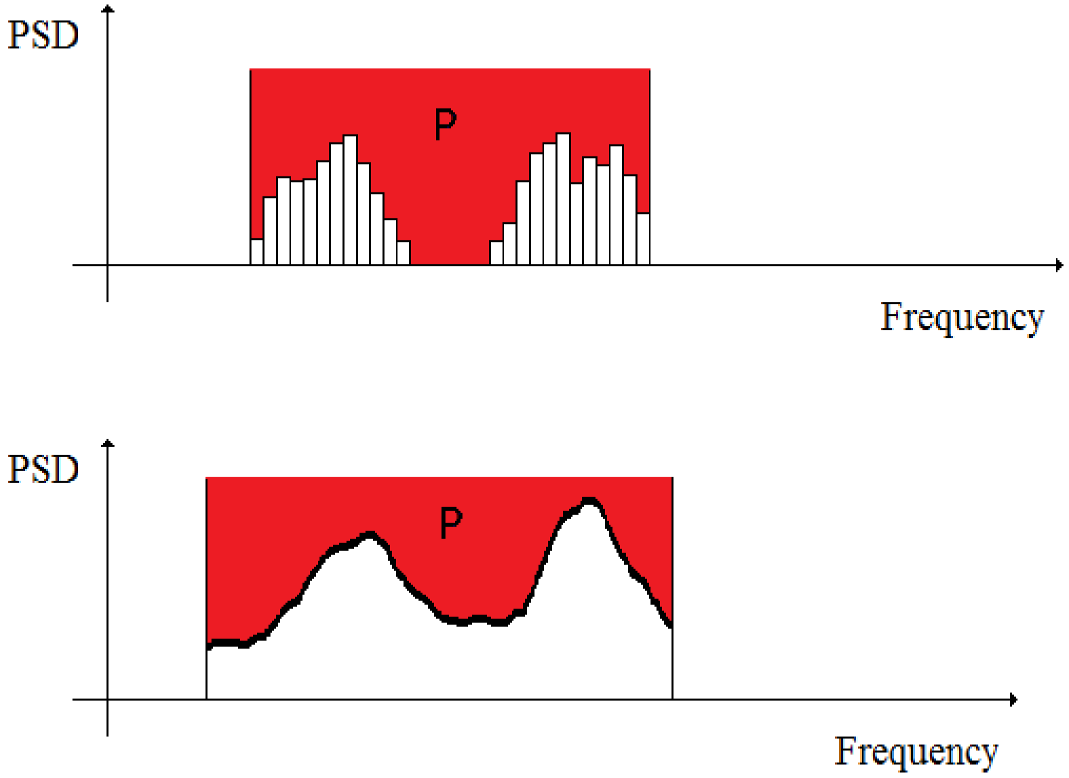
$$\frac{dR_1}{ds} > \frac{dR_2}{ds}$$

$$\frac{B}{\ln 2}\frac{1}{N_1 + P_1} = \frac{B}{\ln 2}\frac{1}{N_2} \Rightarrow$$

$$\frac{dR_1}{ds} = \frac{dR_2}{ds}$$

- We start by investing energy in the first channel until it’s total power becomes equal to N$_2$.
- After that point, energy is divided equally among the two channels.

Water filling strategy

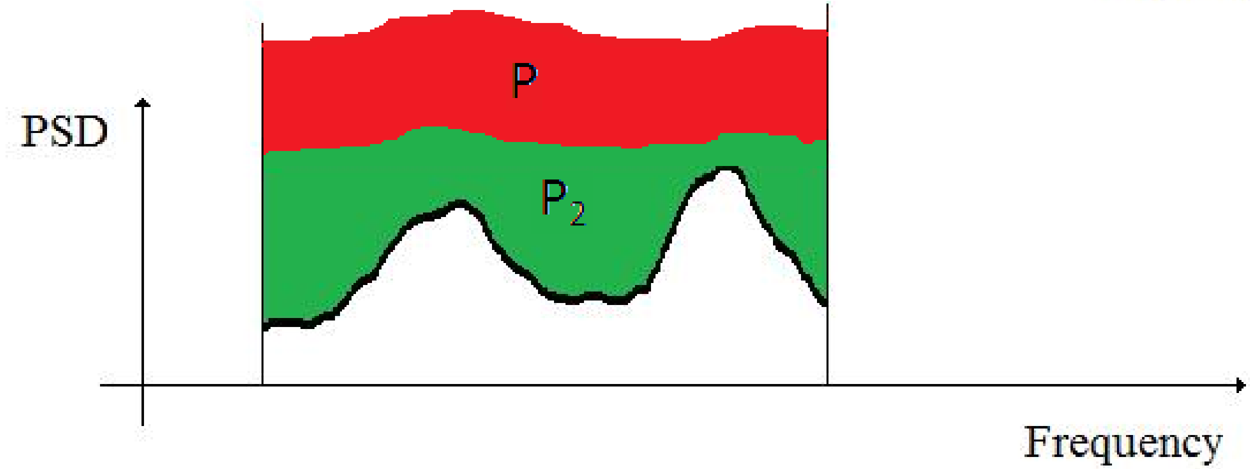


- The best way for a user to invest it’s power is to distribute it in the whole range of frequencies.

Interference between users

- Consider again that 2 systems compete for access in the band [F_1 F_2].
  - According to the water filling strategy both will invest their energy in the whole interval [F_1 F_2].

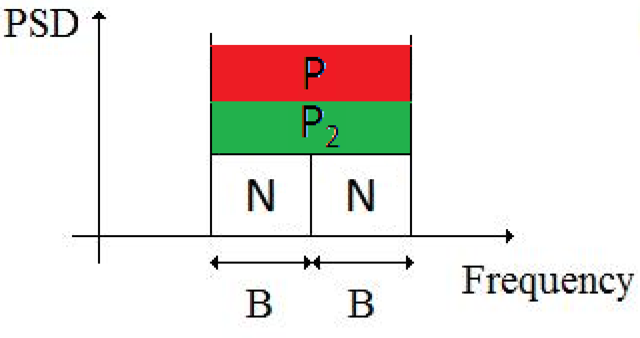
- The first user will achieve a lower rate than expected due to the interference of the second user.

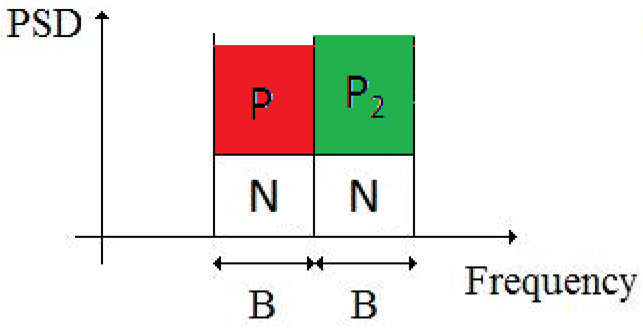


Cooperation

- Is it possible for the two users to achieve a better rate if they cooperate?

- Example:





$$R_1 = 2B\log(1+\frac{P}{P_2+2N})$$

$$R_1' = B\log(1+\frac{P}{N})$$

- When $R_1' > R_1$ then dividing the bandwidth among the two users is more effective than water filling.

Channel allocation problem

- M users compete to access a band.
  - They do not use the selfish water filling strategy
  - Instead they cooperate and divide the spectrum among them in the most efficient way.

- The initial band is divided into a number of non overlapping frequency bins.
  - An algorithm maps the bins to users in such a way that a global utility function is maximized.

Channel allocation algorithm

- There are various ways that a channel allocation algorithm could be designed.
  - Distributed or centralized.
  - Proactive or on demand.
  - Predetermined channel allocation.
  - Allocation of contiguous or non contiguous bins to devices.

Centralized algorithms

- One entity is responsible for the division of channels among users.

- This entity should be periodically informed about various parameters such as:
  - Traffic demand of users
  - Possible changes in the network topology
  - Quality of links

- The amount of information maintained by the centralized entity gets larger as the network grows.
  - Scalability issue

Distributed algorithms

- Each node should be kept informed about the conditions in it’s own neighborhood.
  - If two nodes decide to use a channel they first inform their neighbors for this action.
  - That way no other node interferes with their communication.
  - Each node should be able to store an amount of information in it’s memory.
  - A large number of messages should be exchanged for the algorithm to function.

- Distributed approaches ensure the scalability of the network better than centralized approaches.

Comparison

- Centralized approaches are a better choice for infrastructure networks.
  - The topology of such networks does not change very often.
  - There is an entity with which can maintain the information needed to administrate the network.

- Distributed approaches are more suitable for ad-hoc networks.
  - These networks are usually formed by nodes with limited resources.
  - Scale in an unpredicted way.

Proactive or on demand algorithms

- In proactive approaches, channels are allocated to users periodically.

- On demand approaches allocate channels to users only when they need them.
  - The channel allocation algorithm should be executed more times than in periodic approaches (when the traffic demand is high).
  - Better utilization of spectrum can be achieved.

Predetermined channel allocation

- Channels are allocated to users only when there is a change in the topology.
  - Each user gets an equal share of the bandwidth.
- Due to variation of load throughout the network, some users could need more bandwidth than other at certain times.
  - Users could borrow channels form their neighbors when they need them.

Primary and secondary channels

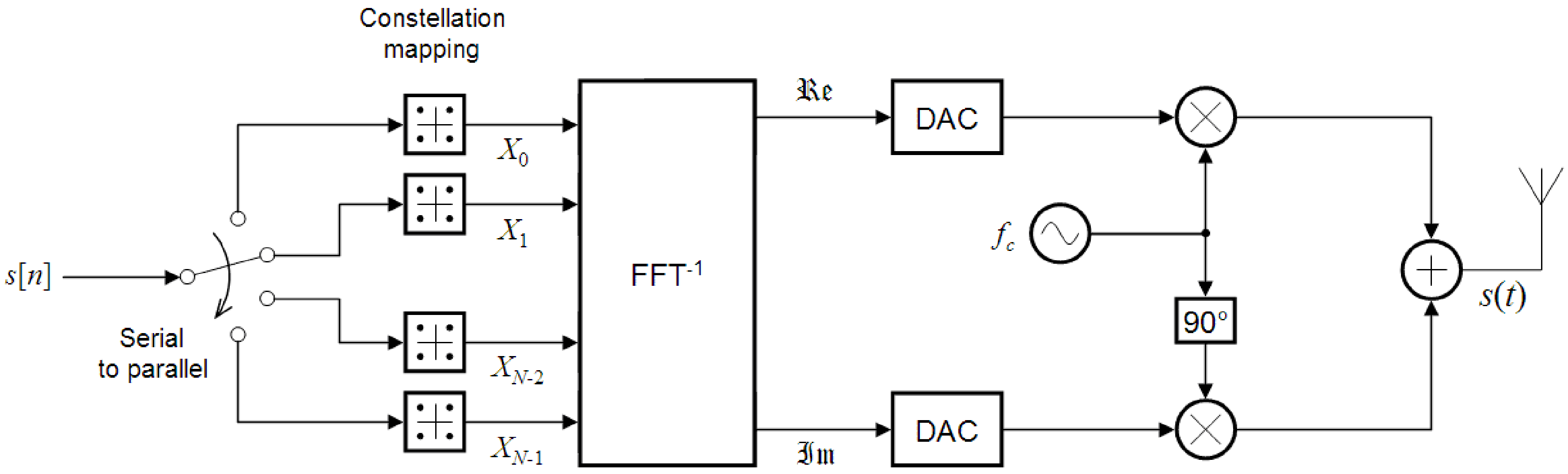
- Channels that are allocated to a user are called **primary**.
- Channels that a user borrows from the neighborhood are called **secondary**.
- Predetermined channel allocation is not so suitable for cognitive radio networks, duo to:
  - Changes of channel conditions caused by primary user activity
  - Network topology changes very often.

Use of contiguous or non contiguous bins

- Is it possible for the channel allocation algorithm to map bins that are not contiguous to a particular user.

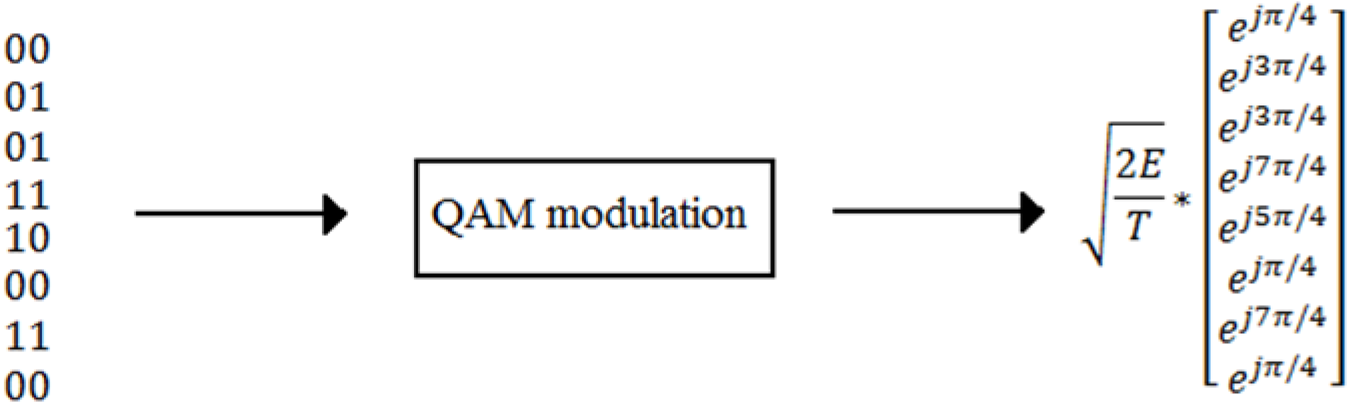
- Answer: Yes, there is a modulation scheme called NC-OFDM that can be used in such a case.

OFDM modulation



- The bit stream is divided into N parallel subflows.
- The symbols of each subflow are modulated using MPSK or MQAM.
- Resulting complex numbers are fed to a module that performs FFT^{-1}.
- Finally the signal is converted from digital to analog, brought to the RF frequencies and then fed to the antenna of the transmitter.

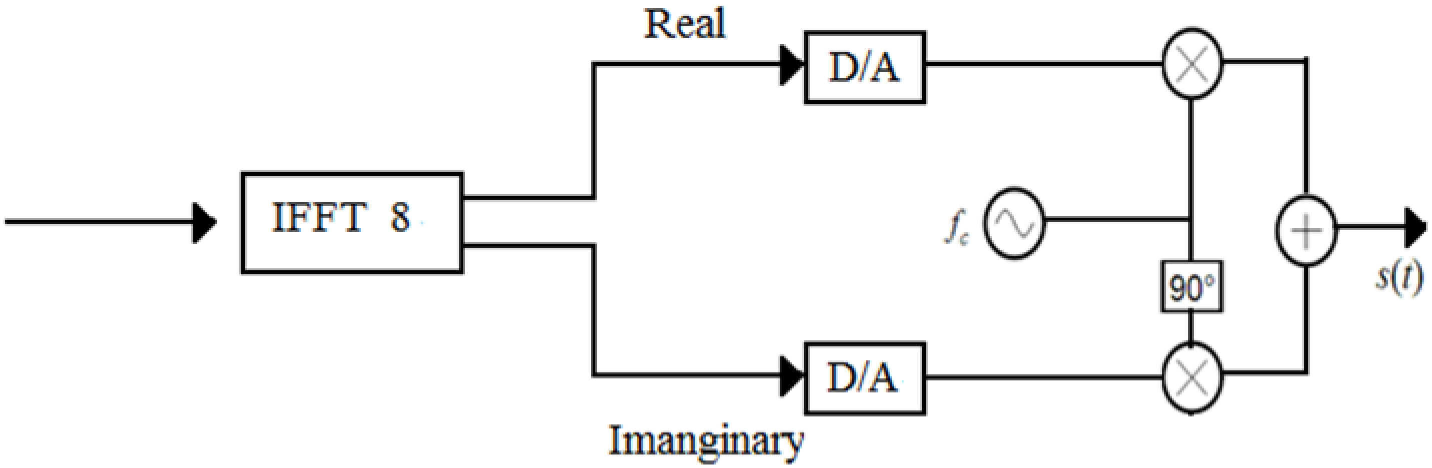
Example modulation



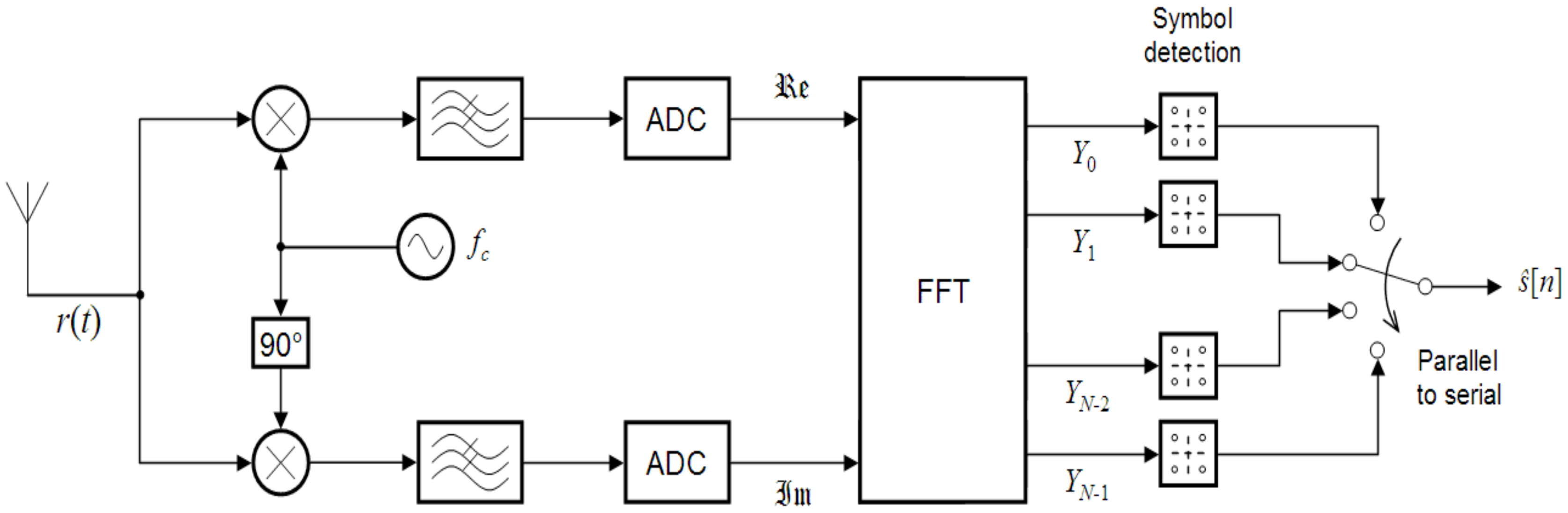
00
01
01
11
10
00
11
00

QAM modulation

$$\sqrt{\frac{2E}{T}} * \begin{bmatrix} e^{j\pi/4} \\ e^{j3\pi/4} \\ e^{j3\pi/4} \\ e^{j7\pi/4} \\ e^{j5\pi/4} \\ e^{j\pi/4} \\ e^{j7\pi/4} \\ e^{j\pi/4} \end{bmatrix}$$

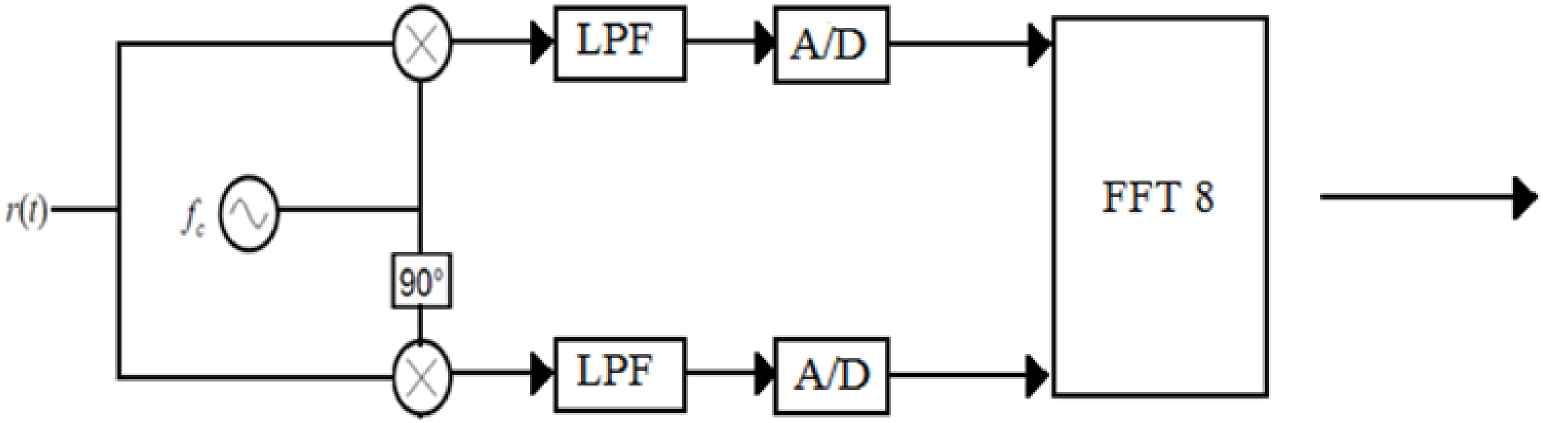


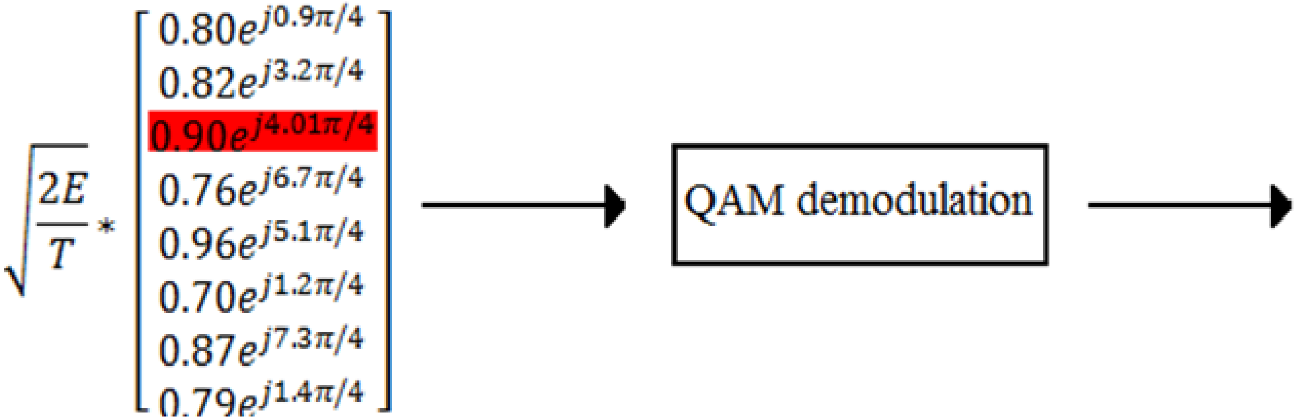
OFDM Demodulation



- At the receiver the inverse procedure is followed.
- First the signal is brought down to baseband and is converted from analog to digital. Then FFT is performed which produces the estimations of the transmitted symbols.

Example demodulation





$$\sqrt{\frac{2E}{T}} * \begin{bmatrix} 0.80e^{j0.9\pi/4} \\ 0.82e^{j3.2\pi/4} \\ 0.90e^{j4.01\pi/4} \\ 0.76e^{j6.7\pi/4} \\ 0.96e^{j5.1\pi/4} \\ 0.70e^{j1.2\pi/4} \\ 0.87e^{j7.3\pi/4} \\ 0.79e^{j1.4\pi/4} \end{bmatrix}$$

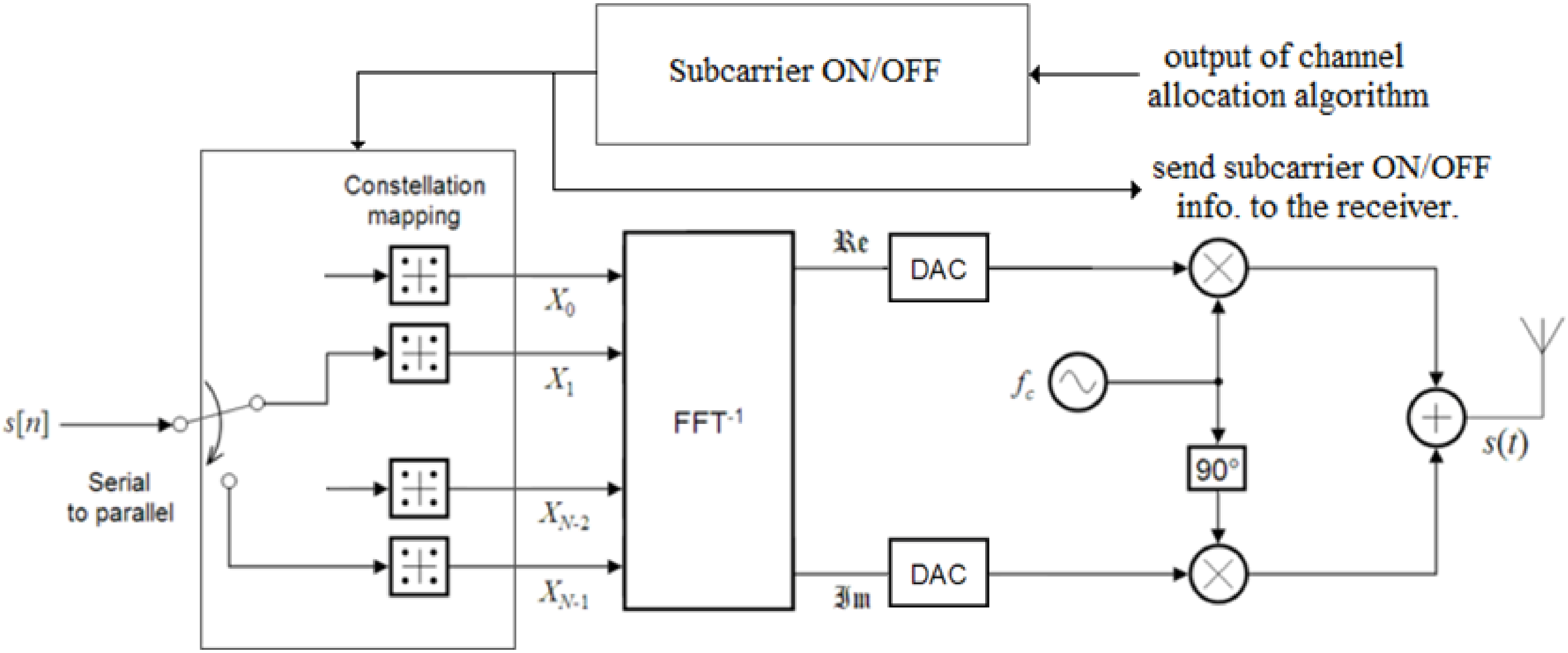
QAM demodulation

	Transitted seq.
00	00
01	01
10	01
11	11
10	10
00	00
11	11
00	00

NC OFDM

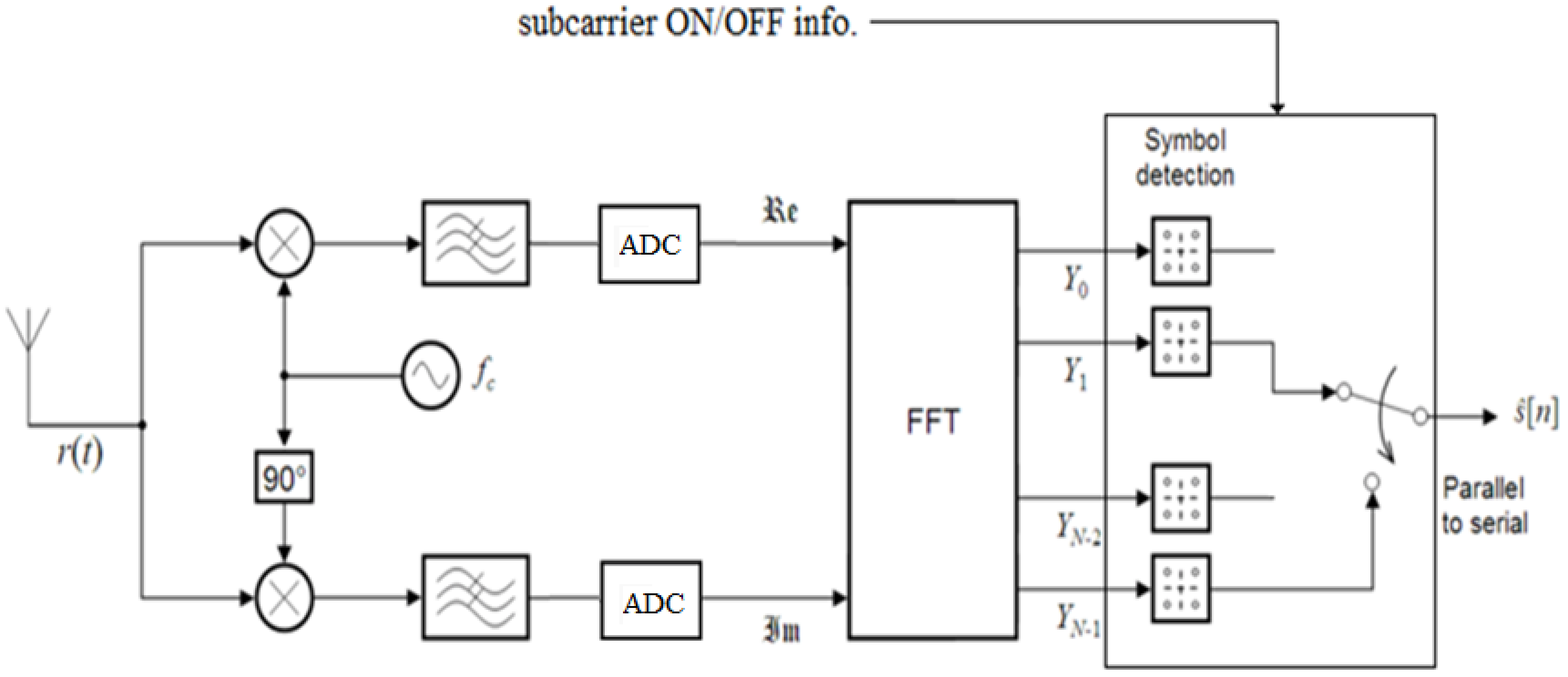
- NC OFDM (non contiguous OFDM) is exactly the same as OFDM with the following deference:
  - Bins that are not allocated to a particular device are deactivated.

NC OFDM TRANSMITTER

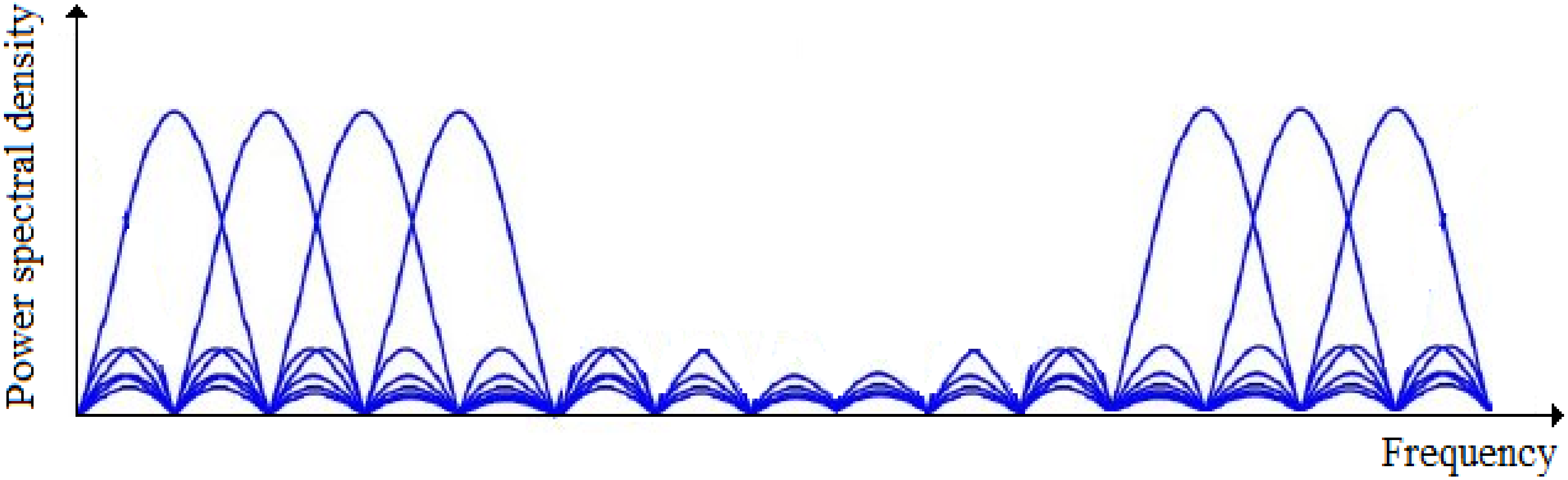


NC OFDM receiver

- At the NC OFDM receiver the reverse process is followed in order to extract the transmitted symbols.



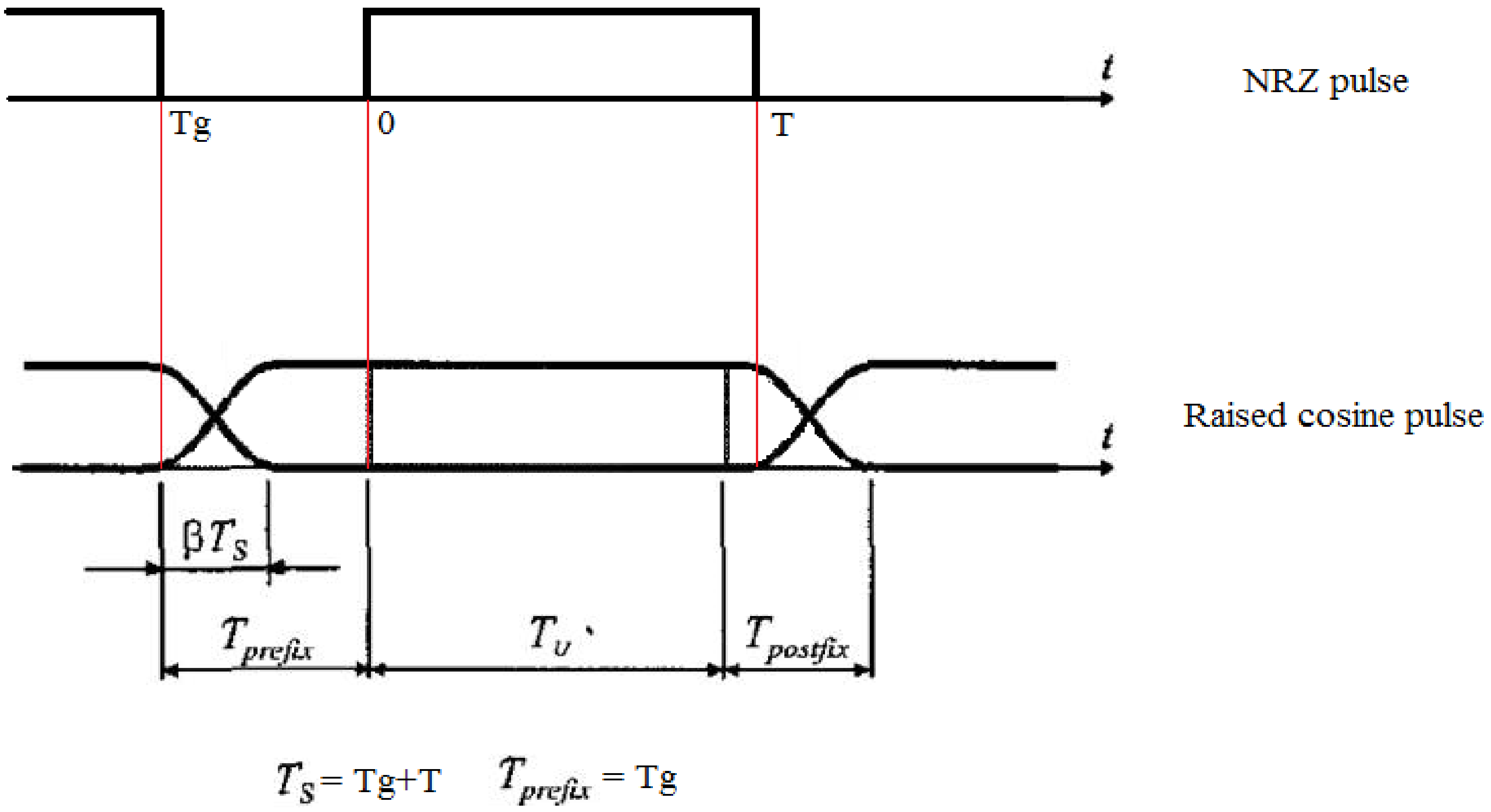
NC OFDM introduces interference



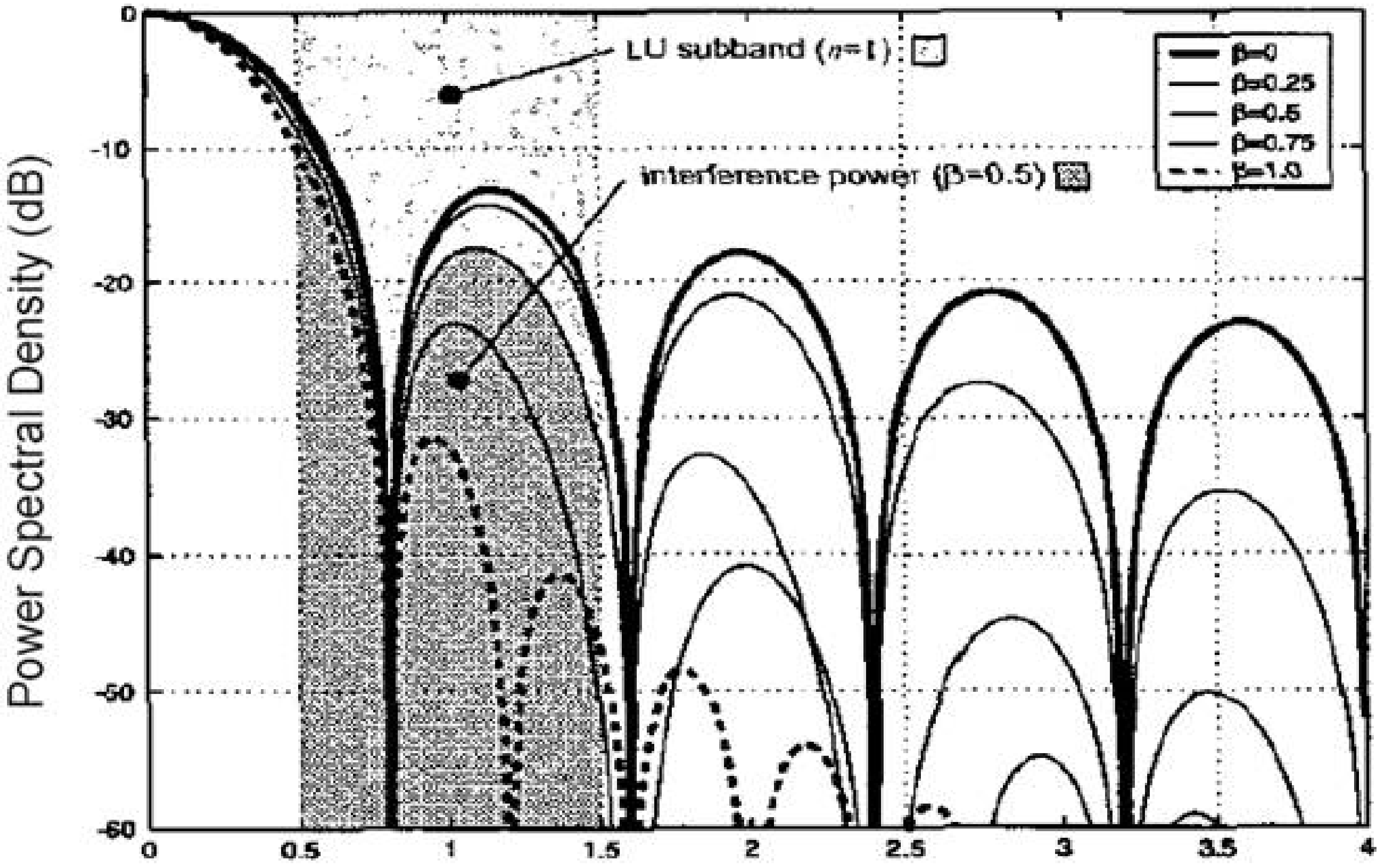
- The NC OFDM modulation scheme introduces a significant amount of interference power to adjacent frequency bins.

Solution 1: windowing of time signal

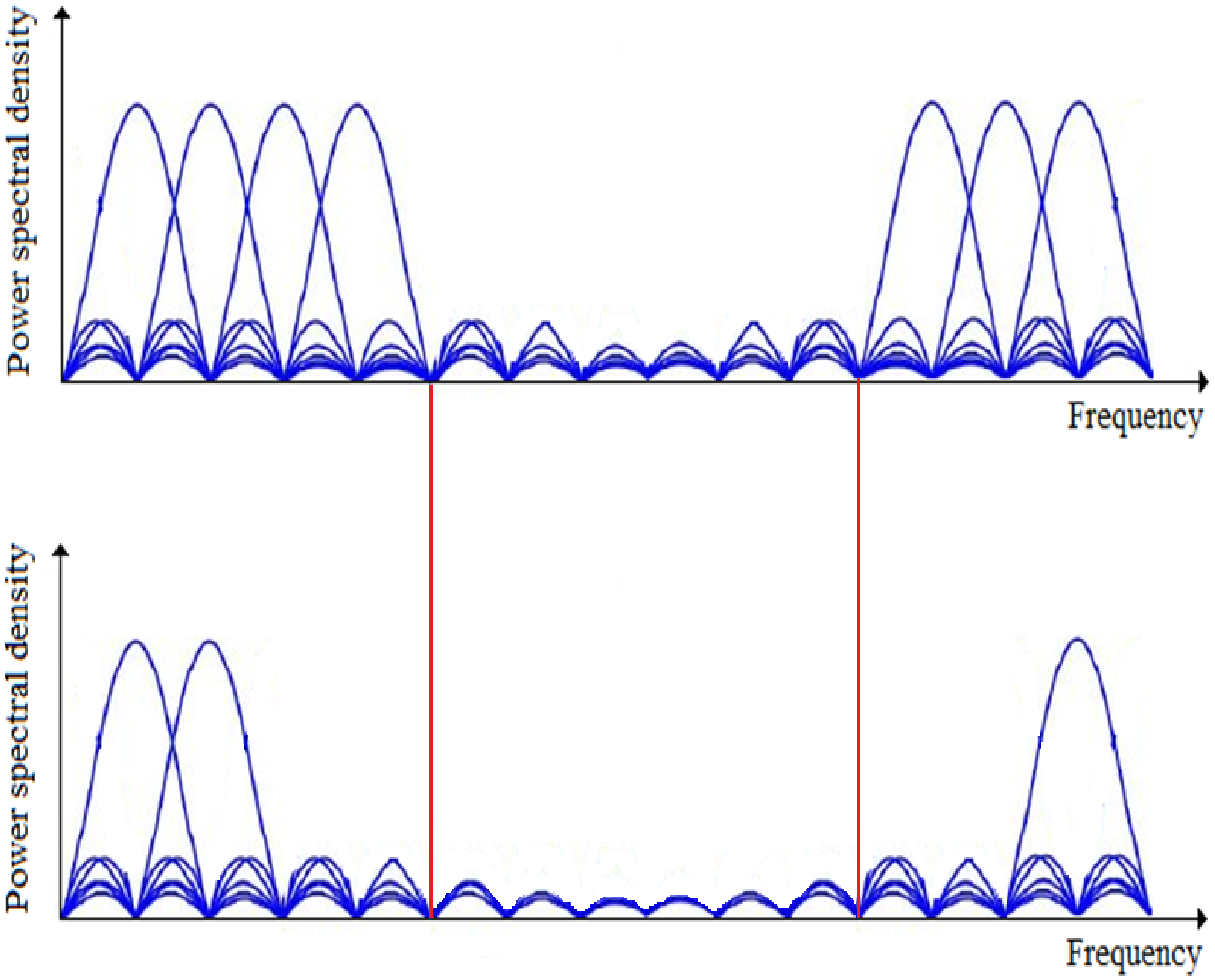
- Use raised cosine pulses for the modulation of the baseband signal instead of NRZ pulses.



Power spectral density of raised cosine pulse



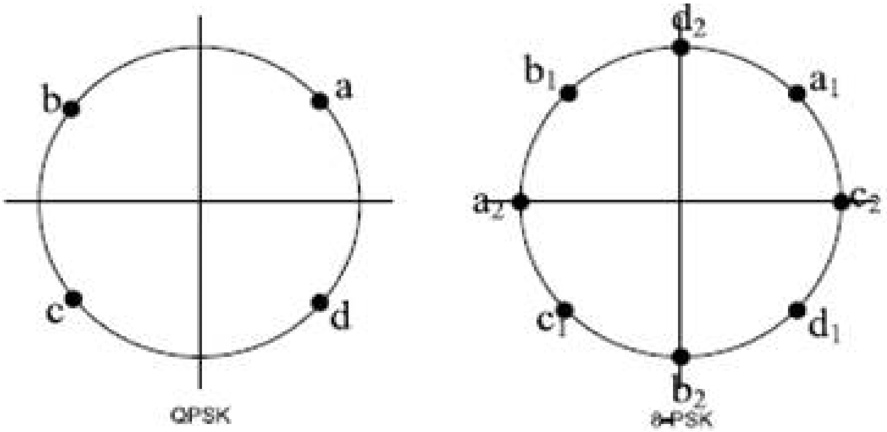
Solution 2: Deactivate some bins at the edges of a frequency zone



- Drawback: large portion of the bandwidth remains unutilized.

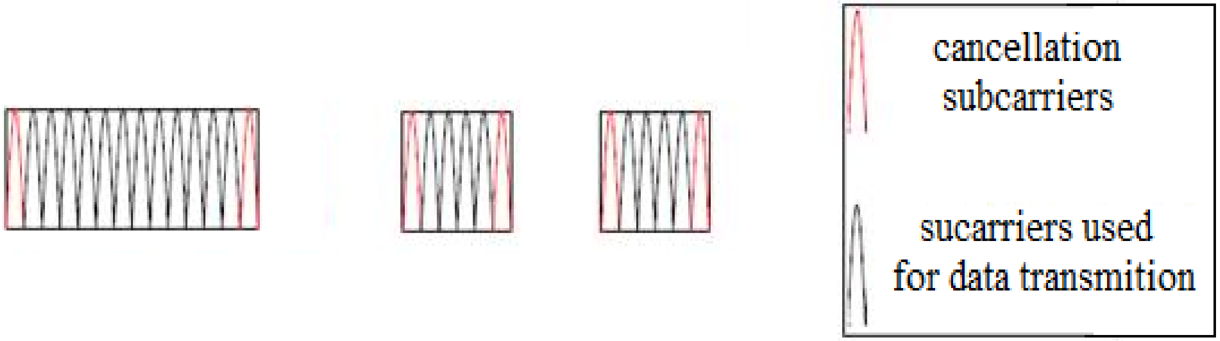
Solution 3: Constellation expansion

- The signal constellation is mapped to another constellation such that:
  - Each symbol corresponds to N (usually 2) points at the new constellation.



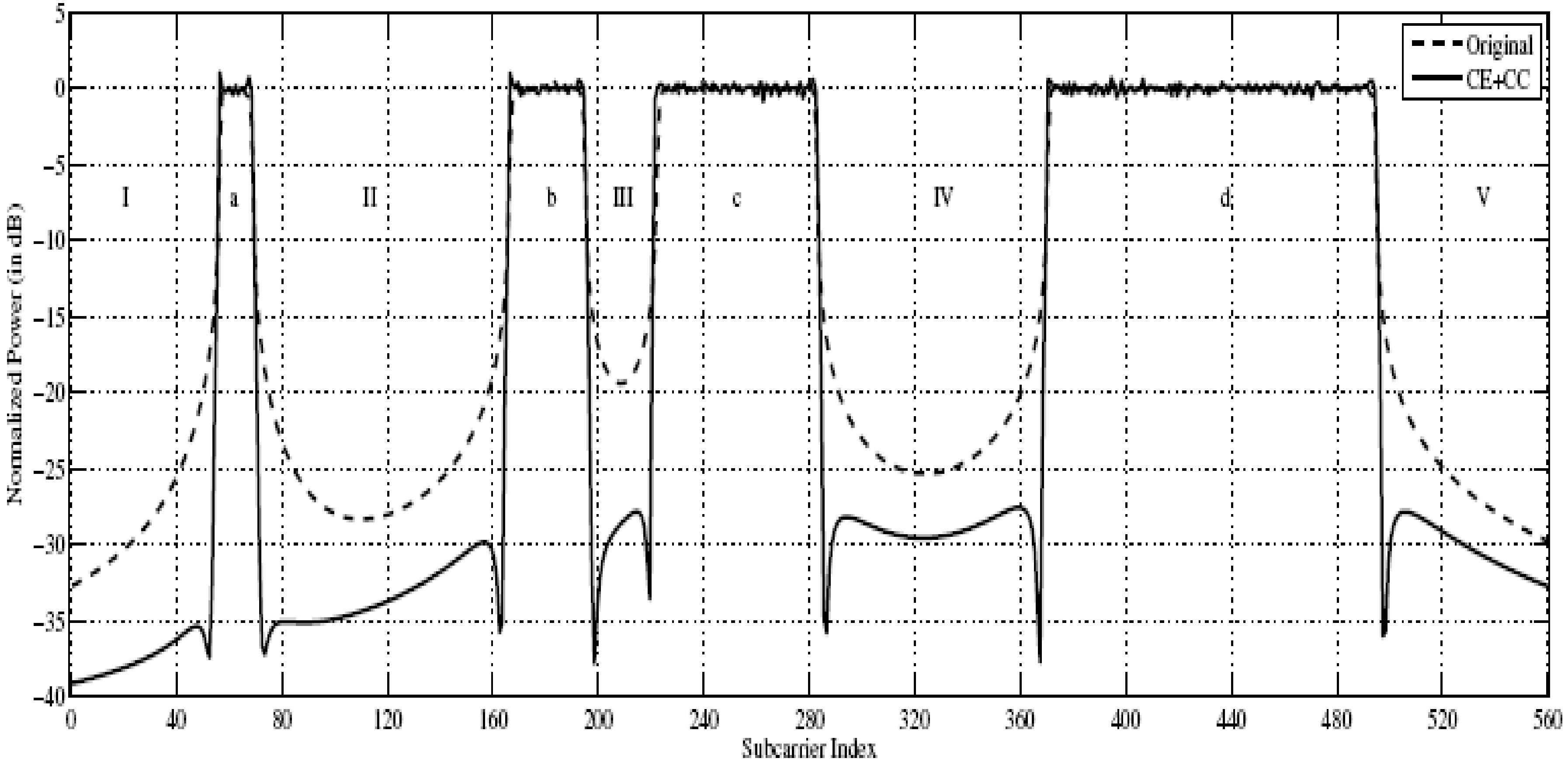
- If we take a sequence of k symbols we can represent it with N^k different ways.
  - We choose the way that reduces the sidelobe power levels.

Solution 4: Cancellation subcarrires



- We use one or two bins at the edges of all frequency zones that are allocated to a device and modulate them, such that:
  - The resulting signal is the opposite of the sidelobe signal.
- Drawbacks
  - A part of the transmission power is spend to modulate the CCs.
  - A portion of the available bandwidth remains unutilized.

Combined use of constellation expansion and cancellation subcarriers



References 1/2

- Channel allocation problem:
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- *Centralized and periodic channel allocation*
  - T. Moscibroda, R. Chandra, Y. Wu, S. Sengupta, and P. Bahl. “Load-aware spectrum distribution in wireless LANs”. In *ICNP’08.*
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  - Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu. “Allocating Dynamic Time-Spectrum Blocks in Cognitive Radio Networks”. In *Proc. of MOBIHOC, 2007.*

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- NC-OFDM:
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- Predetermined channel allocation:
  - K. Xing, X. Cheng, L. Ma, and Q. Liang. Superimposed code based channel assignment in multi-radio multi-channel wireless mesh networks. In *MobiCom '07.*
  - A. Vasan, R. Ramjee, and T. Woo. “ECHOS: Enhanced Capacity 802.11 Hotspots”. In Proceedings of IEEE INFOCOM 2005.

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Part two

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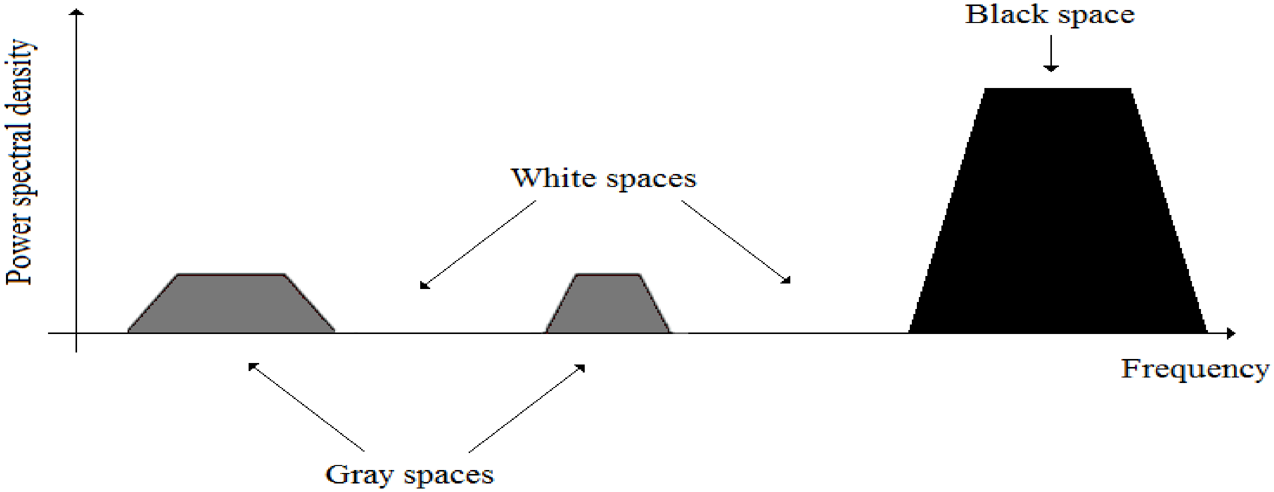
2. Interference cancellation

- Black space: a portion of the spectrum in which the primary user's signal is very strong.



- Is there a way for a secondary system to function in a black space?
  - Use an interference cancellation technique.

Key innovation

- The idea is to find a way to estimate the primary user’s signal at the secondary receiver.
    - Subtract this estimation from the overall signal.
    - That way a significant amount of interference power would be cancelled.

- The secondary user’s signal can now be decoded under a much higher value of SINR.

Decode the primary signal

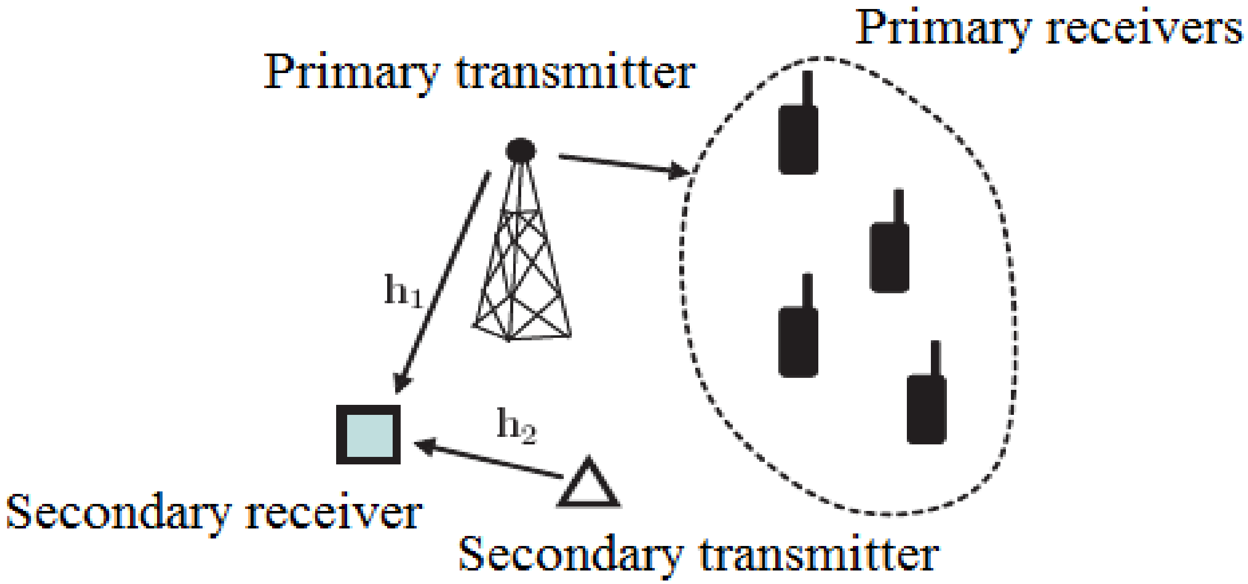
- The simplest way to estimate the primary signal is to decode it.

- For such a purpose the secondary receiver should know the primary user’s modulation scheme.
  - This information is assumed to be broadcasted by the primary user.

- Also the secondary receiver should be equipped with the proper hardware to perform the demodulation procedure.

Problem formulation

- A primary and a secondary system function at the same region.
  - The width of the band that is used by these systems is denoted by B.



Some definitions

- The secondary receiver observes an overall signal that consists of the following components:
  1. The primary system's signal of power P
  2. The secondary system's signal of power S
  3. The noise signal of power N.

- If we use the notation $\gamma_s = \frac{S}{N}$ and $\gamma_p = \frac{P}{N}$ then the values of SINR for the secondary and the primary signal are:

$$SINR_s = \frac{S}{P+N} = \frac{S/N}{1+P/N} = \frac{\gamma_s}{1+\gamma_p} \qquad SINR_p = \frac{P}{S+N} = \frac{P/N}{1+S/N} = \frac{\gamma_p}{1+\gamma_s}$$

SINR requirement

- If the primary transmitter uses the rate R_p then it's signal can be decoded only if $SINR_p > \beta_p$, where:

$$R_p = B\log(1+\beta_p)$$

- In other words β_p is the **minimum** value of SINR that is required for successful decoding of the primary signal.

- We will distinguish the following two cases:
  1. $SINR_p > \beta_p$
  2. $SINR_p < \beta_p$

1. $SINR_p > \beta_p$

- In this case the primary signal is decoded and subtracted from the overall signal.
  - Only the secondary signal and noise remains.

- The value of SINR for the secondary signal becomes now:

$$SINR_s^{'} = \frac{S}{N} = \gamma_s$$

- This means that the achievable rate for the secondary system is:

$$R_s^{'} = B\log(1+\gamma_s)$$

2. $SINR_p < \beta_p$

- We again distinguish two subcases:

- $\gamma_p < \beta_p$: Even if the secondary signal was absent it would still be impossible to decode the primary signal.
  - The achievable rate for the secondary system is:

$$R_s = B\log(1 + SINR_s) = B\log\left(1 + \frac{\gamma_s}{1+\gamma_p}\right)$$

- $\gamma_p > \beta_p$: We can use a method called superposition coding to achieve a better rate than R_s.

Superposition coding 1/2

- The secondary transmitter sends two streams of information denoted by x_1 and x_2.
  - The first stream uses a portion α of the transmission power.
  - The remaining power is used for the modulation of the second stream.
- Define as β_{s1} and β_{s2} the minimum value of SINR that is required for successful decoding of signals x_1 and x_2. If:

$$\frac{a\gamma_s}{1+\gamma_p+(1-a)\gamma_s} \geq \beta_{s1}$$

- The first stream can be decoded and subtracted from the overall signal.
  - Only the signal of the second stream, the primary signal and noise will remain.

Superposition coding 2/2

- Now the value of SINR for the primary signal has changed into:

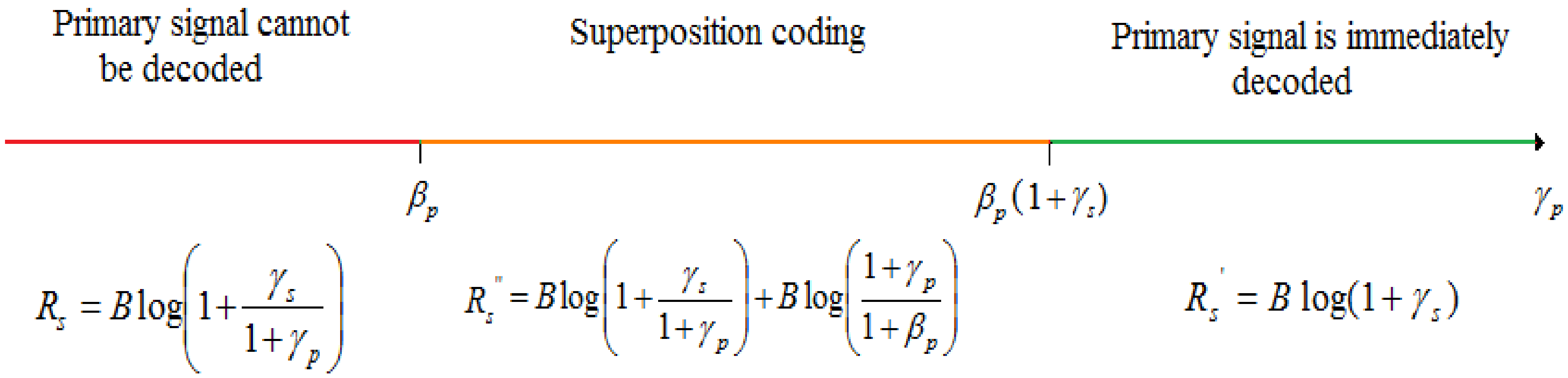
$$SINR_p{}' = \frac{\gamma_p}{1+(1-\alpha)\gamma_s}$$

- We can choose α such that $SINR_p{}' \geq \beta_p$. Now the primary signal can be decoded.
  - Only the second stream and noise will remain.

- The achievable rate for the secondary system is:

$$R_s{}'' = B\log\left(1+\frac{\alpha\gamma_s}{1+\gamma_p+(1-\alpha)\gamma_s}\right)+B\log(1+(1-\alpha)\gamma_s) = B\log\left(1+\frac{\gamma_s}{1+\gamma_p}\right)+B\log\left(\frac{1+\gamma_p}{1+\beta_p}\right)$$

Summary



Primary signal cannot be decoded

$$R_s = B\log\left(1+\frac{\gamma_s}{1+\gamma_p}\right)$$

Superposition coding

$$R_s'' = B\log\left(1+\frac{\gamma_s}{1+\gamma_p}\right) + B\log\left(\frac{1+\gamma_p}{1+\beta_p}\right)$$

Primary signal is immediately decoded

$$R_s' = B\log(1+\gamma_s)$$

- Using the interference cancellation technique we can achieve much higher data rates.
- It is better that the primary signal’s power is high.
  - That way it can be estimated more accurately.

3. Adaptive modulation

- Consider that a pair of nodes communicate using a channel of width B and transmission power equal to P.

- According to Shannon the capacity of the channel is:

$$C = B\log(1+\gamma)$$

- Where γ denotes the value of SNR at the receiver.

Fading channel

- If the channel is affected by fading phenomena the value of γ will vary according to a PDF p(γ) which is:
  - Lognormal if the dominant fading phenomenon is shadowing.
  - Exponential if multipath fading is dominant (Rayleigh fading).

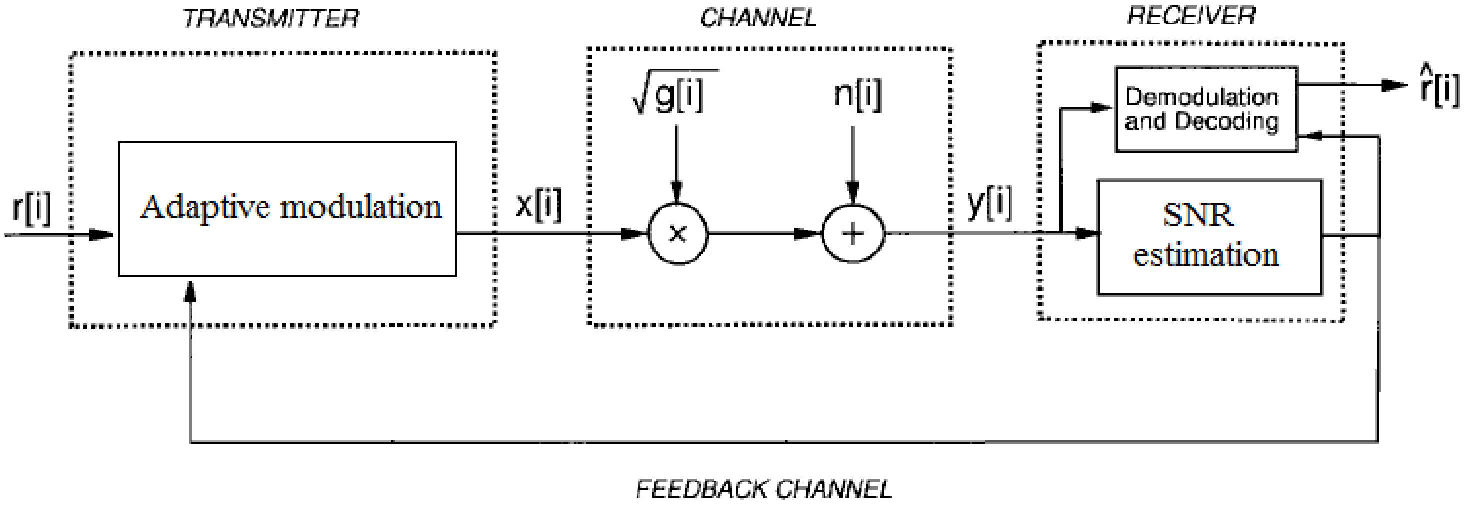
- We could now define the mean channel capacity as:

$$C_m = \int_0^\infty B \log(1+\gamma)\, p(\gamma)\, d\gamma$$

- This is a theoretical result and we do not know a practical method to achieve it in real networks.

Problem formulation

- According to the current value of γ decide which is the best modulation scheme to use, in order to maximize the throughput.
- The value of γ is estimated at the receiver and sent to the transmitter through a control channel.



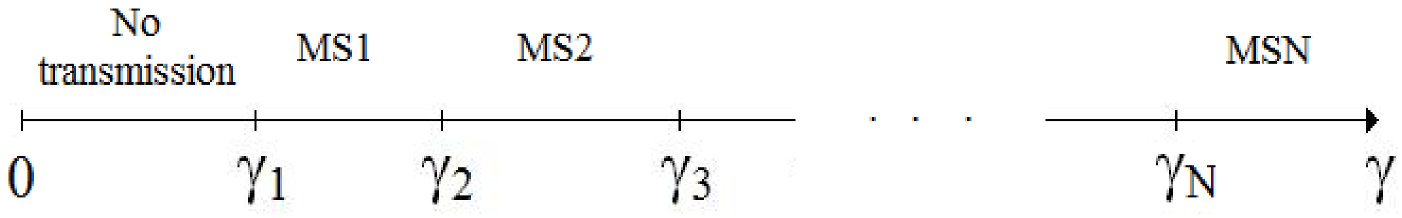
Hardware limitations

- If the transmitter was able to change it's rate in a continuous manner then throughput would be close to capacity.

- Due to hardware limitations the transmitter has to choose among a **limited** number of modulation schemes.
  - The transmission rate could also take a finite number of different values.

Partition of SNR space

- Assume that the transmitter can use N different modulation schemes.
  - We can partition the space of possible values of SNR into N+1 non overlapping regions.



- If SNR$<\gamma_1$ the channel condition is poor and no transmission is performed.
- If $\gamma_1<$SNR$<\gamma_2$ the first modulation scheme is used.
- If $\gamma_2<$SNR$<\gamma_3$ the second modulation scheme is used etc.

Objective

- Our goal is to determine the values of $\gamma_1, \gamma_2, \ldots, \gamma_n$ such that the throughput is maximized.

- Because the number of modulation schemes is finite, the achievable throughput will be less than the capacity.

- An increase in the number of available modulation schemes yields better approximations of the capacity.
  - Modulation schemes should change more quickly in this case.

4. Power control

- Power control is a method that is used to increase the value of SINR if it is too low or decrease it if it is too high.
  - This can be done by appropriate adjustment of transmission powers.

- In other words the goal of power control is to minimize the overall power that is needed in order to satisfy the SINR requirements of all links within a network.

Problem formulation

- Consider a set of M transmitter-receiver pairs that share the same channel.
  - G_{ij} : Link gain between transmitter i and receiver j.
  - P_i : Transmission power if the ith transmitter.
  - $G_{ji}P_j$: Power of the signal of the jth transmitter at the ith receiver.

- The transmitter i communicates with the receiver i.
  - The desired signal at receiver i is equal to $G_{ii}P_i$.
  - The interference from other transmitters to receiver i is:

$$I_i = \sum_{j \neq i} G_{ji} P_j$$

SINR conditions

- The value of the SINR at the ith receiver is expressed as:

$$\Gamma_i = \frac{G_{ii} P_i}{\sum_{j \neq i} G_{ji} P_j + N_i}$$

Where N_i is the power of noise.

- To ensure the successful communication of all transmitter-receiver pairs the following conditions should be satisfied:

$\Gamma_i \geq \gamma_0$ for each i = 1, 2, ..., M

Conditions in matrix form

- We can write the SINR conditions in matrix form as follows:

$$[\mathbf{I} - \gamma_0 \mathbf{F}]\mathbf{P} \geq \mathbf{u}$$

- Where:
  - **P** = $[P_1\ P_2\ \ldots\ P_M]^T$ is the transmission powers vector.
  - **u** is a vector with elements $u_i = \gamma_0 N_i / G_{ii}$.
  - **F** is a matrix defined as:

$$F_{ij} = \begin{cases} 0 & \text{if } j = i \\ G_{ji} / G_{ii} & \text{if } j \neq i \end{cases}$$

Formulation as optimization problem

- The power control problem can now be formally defined as follows:

$$\text{minimize } \sum_i P_i$$

$$\text{subject to } [\mathbf{I}\text{-}\ \gamma_0\mathbf{F}]\mathbf{P} \geq \mathbf{u}$$

- If the matrix $[\mathbf{I} - \gamma_0\mathbf{F}]$ is positive definite then the solution of the above problem is the following:

$$\mathbf{P}_{opt} = [\mathbf{I} - \gamma_0\mathbf{F}]^{-1}\mathbf{u}$$

5. Beamforming

- Consider that at the receiver of a secondary system there is an array of M antennas.
  - The outputs of the array elements are multiplied by a weight factor and are added in order to construct the received signal.

- By varying the weight factors we can adjust the beampattern of the receiver.
  - That way we could place nulls at the directions of interfering sources and the main lobe at the direction of the signal of interest.

Problem formulation

- We consider a set of M transmitter and receiver pairs that function at the same channel.
  - Each receiver uses an antenna array with K elements.
  - The gain of the ith array at the direction of arrival θ is defined as:

$$\mathbf{v}_i(\theta) = [v_i^1(\theta) \quad v_i^2(\theta) \quad \ldots \quad v_i^{\mathrm{K}}(\theta)]^T$$

- Where $v_i^k(\theta)$ is the gain of the kth antenna element of the ith receiver at the direction θ.

Received signal

- The received signal at the ith receiver is defined as follows:

$$x_i(t) = \sum_{j=1}^{M} \sqrt{P_j G_{ji}} \sum_{l=1}^{L} a^{l}{}_{ji} \mathbf{v}_i(\theta_l) s_j(t-\tau_j) + \mathbf{n}_i(t)$$

- Where:
  - S_j (t) is the message signal of the jth transmitter.
  - τ_j is a time delay that corresponds to the arrival of the message signal at the receiver.
  - $\mathbf{n}_i(t)$ is the thermal noise vector.
  - P_j is the power of the jth transmitter.
  - a^{l}_{ji} is the attenuation due to shadowing at the lth path.

- To simplify the above equation we set:

$$\boldsymbol{\alpha}_{ji} = \sum_{l=1}^{L} \alpha^{l}{}_{ji} v_i(\theta_l)$$

Beamforming objectives

- The output of the ith antenna array can be written as follows:

$$e_i(n) = \mathbf{w}_i^H \mathbf{x}_i(nT)$$

Where $\mathbf{w}_i$ is a vector that contains the weights with which we multiply the output of each antenna element.

- Goals:
  - Minimize the average output power $\varepsilon_i = E\{\mathbf{w}_i^H \mathbf{x}_i(nT)\, \mathbf{x}_i^H(nT) \mathbf{w}_i\}$.
  - Maintain unity gain at the direction of the desired signal $\mathbf{w}_i^H \mathbf{a}_{ii} = 1$.

Average output power

- By performing some calculations the average output power can be written as follows:

$$\varepsilon_i = \mathrm{E}\{\mathbf{w}_i^H \mathbf{x}_i(nT)\, \mathbf{x}_i^H(nT)\mathbf{w}_i\} = \mathbf{w}_i^H\, \mathrm{E}\{\mathbf{x}_i(nT)\, \mathbf{x}_i^H(nT)\}\mathbf{w}_i = \mathbf{w}_i^H \boldsymbol{\Phi}_i \mathbf{w}_i$$

where:

$$\boldsymbol{\Phi}_i = \sum_{j \neq i} P_j G_{ji} \mathbf{a}_{ji} \mathbf{a}^H{}_{ji} + N_i \mathbf{I} + P_i G_{ii} \mathbf{a}_{ii} \mathbf{a}^H{}_{ii} = \boldsymbol{\Phi}_{in} + P_i G_{ii} \mathbf{a}_{ii} \mathbf{a}^H{}_{ii}$$

and

$$\boldsymbol{\Phi}_{in} = \sum_{j \neq i} P_j G_{ji} \mathbf{a}_{ji} \mathbf{a}^H{}_{ji} + N_i \mathbf{I}$$

Formulation as an optimization problem

- The objectives of beamforming can be written as an optimization problem:

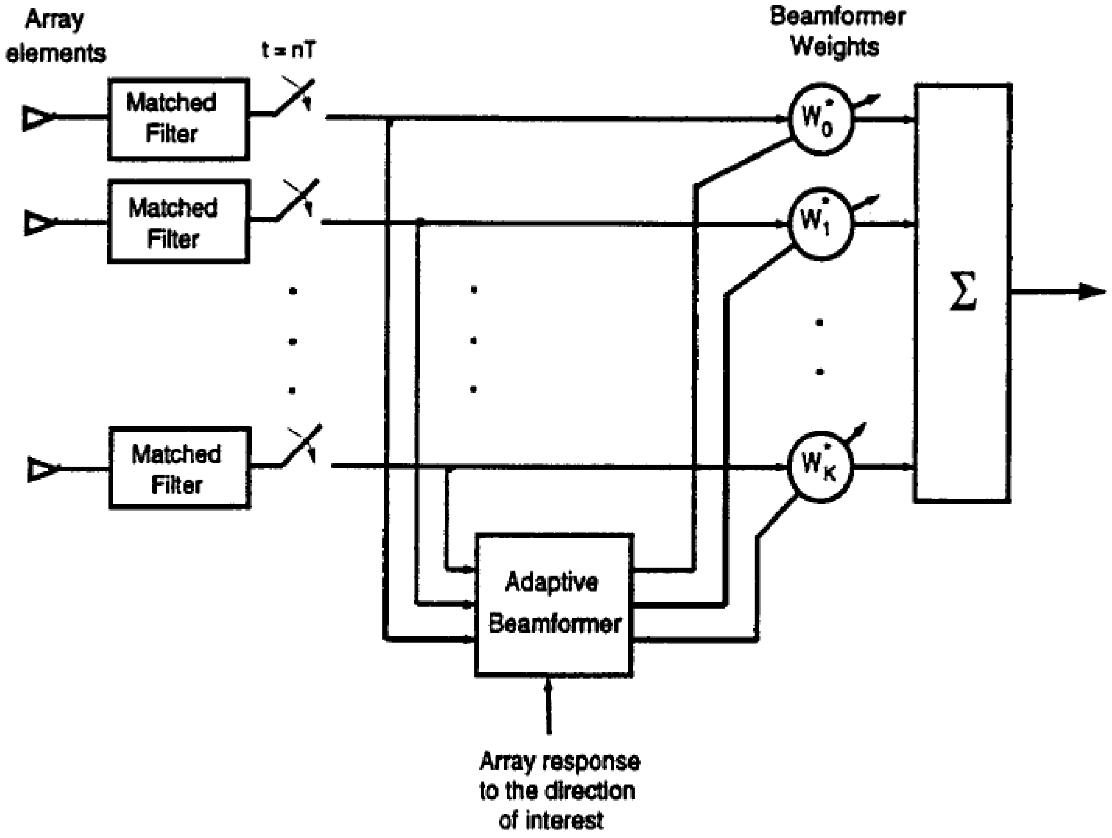
$$\text{minimize} \quad P_iG_{ii} + \sum_{j\neq i} P_jG_{ji}\mathbf{w}^{H}{}_{i}\mathbf{a}_{ji}\mathbf{a}^{H}{}_{ji}\mathbf{w}_i + N_i\mathbf{w}^{H}{}_{i}\mathbf{w}_i$$

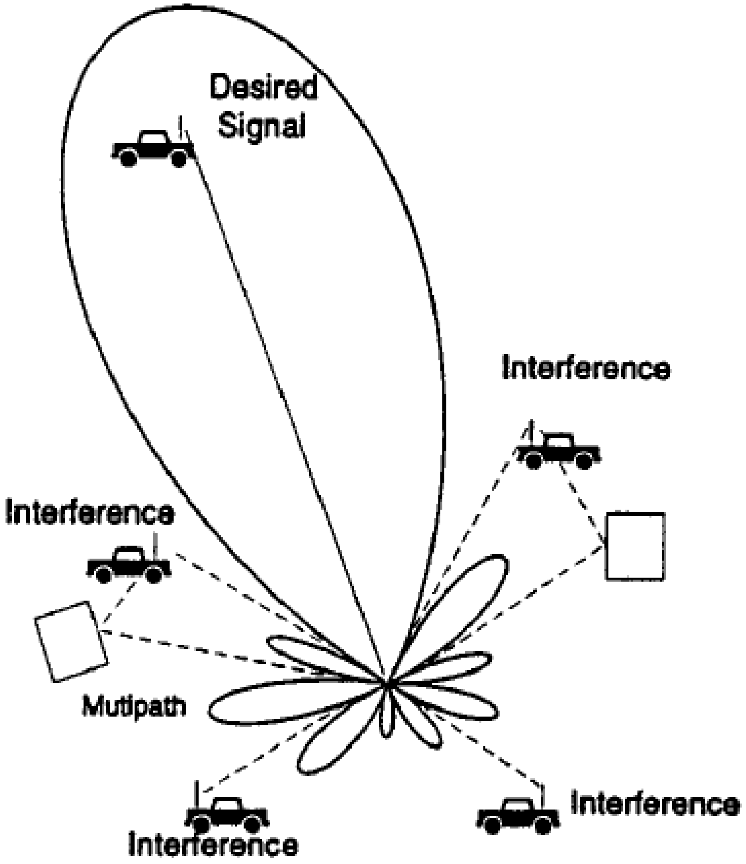
$$\text{subject to} \quad \mathbf{w}_{\mathrm{i}}^{\mathrm{H}}\mathbf{a}_{\mathrm{ii}} = 1$$

- Solution using Lagrange multipliers:

$$\mathbf{w}_{\mathrm{ioptm}} = \frac{\mathbf{\Phi}_{in}{}^{-1}\mathbf{a}_{\mathrm{ii}}}{\mathbf{a}_{\mathrm{ii}}{}^{\mathrm{H}}\mathbf{\Phi}_{in}{}^{-1}\mathbf{a}_{\mathrm{ii}}}$$

Example


Array elements
t = nT
Matched Filter
Matched Filter
Matched Filter
Beamformer Weights
W_0^*
W_1^*
W_K^*
Σ
Adaptive Beamformer
Array response to the direction of interest


Desired Signal
Interference
Interference
Mutipath
Interference
Interference

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  - Popovski, P. and Yomo, H. and Nishimori, K. and Di Taranto, R. and Prasad, R., “Opportunistic Interference Cancellation in Cognitive Radio Systems,” *IEEE International Symposium on New Frontiers in DynamicSpectrum Access Networks, pp. 472–475, April 2007.*

- Adaptive modulation:
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  - F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, “Joint optimal power control and beamforming in wireless networks using antenna arrays,” *IEEE Trans. Commun., vol. 46, pp. 1313–1324, Nov. 1998.*