



**ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ  
ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ**

# **Ψηφιακή Επεξεργασία Φωνής**

**Ενότητα 1η: Ψηφιακή Επεξεργασία Σήματος**

Στυλιανού Ιωάννης

Τμήμα Επιστήμης Υπολογιστών

# CS578- SPEECH SIGNAL PROCESSING

## LECTURE 1: DISCRETE-TIME SIGNAL PROCESSING FRAMEWORK

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# OUTLINE

- 1 DISCRETE-TIME SIGNALS AND SYSTEMS
- 2 DISCRETE-TIME FOURIER TRANSFORM
- 3 Z-TRANSFORM
- 4 LTI SYSTEMS IN THE FREQUENCY DOMAIN
- 5 PROPERTIES OF LTI SYSTEMS
- 6 DISCRETE FOURIER TRANSFORM
- 7 A/D AND D/A

# DISCRETE-TIME SIGNALS

- Unit sample or “impulse”:

$$\begin{aligned}\delta[n] &= 1, & n = 0 \\ &= 0, & n \neq 0\end{aligned}$$

- Unit step:

$$\begin{aligned}u[n] &= 1, & n \geq 0 \\ &= 0, & n < 0\end{aligned}$$

- Exponential sequence:

$$x[n] = A\alpha^n$$

- Sinusoidal sequence:

$$x[n] = A \cos(\omega n + \phi)$$

- Complex exponential sequence:

$$x[n] = Ae^{j\omega n}$$

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# DISCRETE-TIME SYSTEMS

Discrete-time System:

$$y[n] = T\{x[n]\}$$

Important class of systems: Linear and Time Invariant (LTI):

- Linearity:

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\}$$

- Time-Invariant:

$$\begin{aligned} \text{if } y[n] &= T\{x[n]\} \\ \text{then } y[n - n_0] &= T\{x[n - n_0]\} \end{aligned}$$

Important property of LTI:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n - k] \\ &= x[n] \star h[n] \end{aligned}$$

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# STABILITY AND CAUSALITY FOR LTI

Necessary and sufficient conditions for:

- Stability:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

- Causality:

$$h[n] = 0, \text{ for } n < 0$$

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# DISCRETE-TIME FOURIER TRANSFORM, DTFT

Discrete-Time Fourier Transform pair:

- Direct:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Inverse:

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{j\omega n} d\omega$$

Example:

$$Ae^{j\omega_0 n + \phi} \leftrightarrow 2\pi Ae^{j\phi} \delta(\omega - \omega_0)$$



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# DTFT PROPERTIES

- Fourier transform is complex:

$$\begin{aligned}X(\omega) &= X_r(\omega) + jX_i(\omega) \\ &= |X(\omega)|e^{j\angle X(\omega)}\end{aligned}$$

- Fourier transform is periodic with period  $2\pi$ :

$$X(\omega + 2\pi) = X(\omega)$$

- For real valued sequence  $x[n]$ :

$$X(\omega) = X^*(-\omega)$$

- Energy of a signal (Parseval theorem):

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

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# UNCERTAINTY PRINCIPLE

Given a signal  $x[n]$  we define as:

- Duration of the signal:

$$D(x) = \sum_{n=-\infty}^{\infty} (n - \bar{n})^2 |x[n]|^2$$

- Bandwidth of the signal:

$$B(x) = \int_{-\pi}^{\pi} (\omega - \bar{\omega})^2 |X(\omega)|^2 d\omega$$

where

$$\begin{aligned}\bar{n} &= \frac{\sum_{n=-\infty}^{\infty} n |x[n]|^2}{\sum_{n=-\infty}^{\infty} |x[n]|^2} \\ \bar{\omega} &= \frac{\int_{-\pi}^{\pi} \omega |X(\omega)|^2 d\omega}{\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega}\end{aligned}$$

*Uncertainty Principle* states that:

$$D(x)B(x) \geq 1/2$$



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# HILBERT TRANSFORM

For a real signal  $x[n]$ , we form the *analytic signal*:

$$s[n] = s_r[n] + js_i[n]$$

where  $s_r[n] = x[n]/2$  and

$$S_i(\omega) = H(\omega)S_r(\omega)$$

where  $H(\omega)$  is referred to as *Hilbert transform*:

$$\begin{aligned} H(\omega) &= -j \quad 0 \leq \omega < \pi \\ &= j \quad -\pi \leq \omega < 0 \end{aligned}$$

# INSTANTANEOUS AMPLITUDE AND FREQUENCY

The analytic signal may be written as:

$$s[n] = A[n]e^{j\theta[n]}$$

- Instantaneous amplitude:

$$A[n] = |s[n]|$$

- Instantaneous frequency:

$$\omega[n] = \left. \frac{d\theta(t)}{dt} \right|_{t=nT}$$

where

$$\theta(t) = \int_{-\infty}^t \omega(\tau) d\tau$$

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# Z-TRANSFORM

z-Transform pair:

- Direct:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Inverse:

$$x[n] = \frac{1}{2\pi j} \oint_C X(z)z^{n-1} dz$$

Example:

$$a^n u[n] \leftrightarrow \frac{1}{1 - az^{-1}} \quad |z| > |a|$$

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# Z-TRANSFORM: RATIONAL FUNCTIONS

Usually:

$$X(z) = \frac{P(z)}{Q(z)} = Az^{-r} \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_0} (1 - b_k z)}{\prod_{k=1}^{N_i} (1 - c_k z^{-1}) \prod_{k=1}^{N_0} (1 - d_k z)}$$

No repeated poles, no poles outside the unit circle:

$$X(z) = \sum_{k=1}^{N_i} \frac{A_k}{(1 - c_k z^{-1})}$$

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# EIGENVALUES, EIGENFREQUENCIES, AND EIGENFUNCTIONS

If  $x[n] = e^{j\omega_0 n}$ , then

$$y[n] = H(\omega_0)x[n]$$

where  $H(\omega)$  is referred to as *frequency response*:

$$H(\omega) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

while  $H(z)$  is usually referred to as *system function* or *transfer function*

# CONVOLUTION THEOREM

If

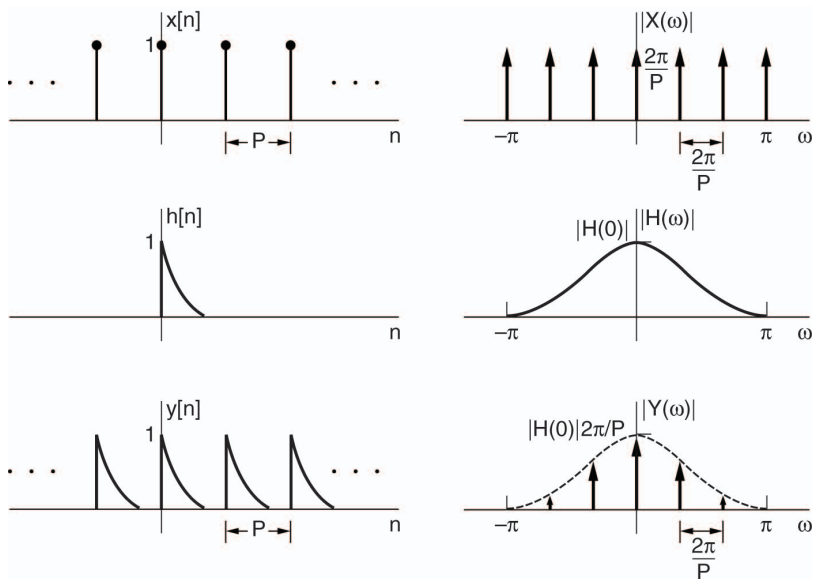
$$x[n] \longleftrightarrow X(\omega)$$

$$h[n] \longleftrightarrow H(\omega)$$

and if:  $y[n] = x[n] \star h[n]$ , then:

$$Y(\omega) = X(\omega)H(\omega)$$

# EXAMPLE OF CONVOLUTION





# WINDOWING (MODULATION) THEOREM

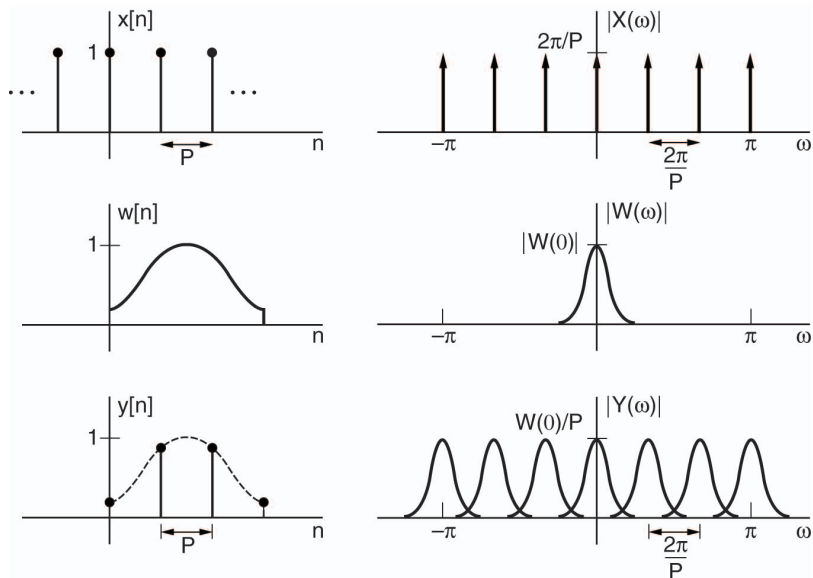
If

$$\begin{aligned}x[n] &\longleftrightarrow X(\omega) \\w[n] &\longleftrightarrow W(\omega)\end{aligned}$$

and if:  $y[n] = x[n]w[n]$ , then:

$$\begin{aligned}Y(\omega) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Theta)W(\omega - \Theta)d\Theta \\ &= \frac{1}{2\pi} X(\omega) \circledast W(\omega)\end{aligned}$$

# EXAMPLE OF MODULATION



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# DIFFERENCE EQUATIONS

In time:

$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

In z-domain:

$$\begin{aligned} H(z) &= \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}} \\ &= A z^{-r} \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_0} (1 - b_k z)}{\prod_{k=1}^{N_i} (1 - c_k z^{-1})} \end{aligned}$$

# MAGNITUDE-PHASE RELATIONSHIPS

- Minimum, Maximum and Mixed-phase systems

$$H(z) = H_{min}(z)H_{max}(z)$$

- Minimum-phase and All-pass system

$$H(z) = H_{min}(z)A_{all}(z)$$

Note that

$$\sum_{n=0}^m |h_{min}[n]|^2 \geq \sum_{n=0}^m |h[n]|^2, \quad m \leq 0$$

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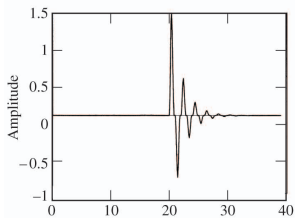
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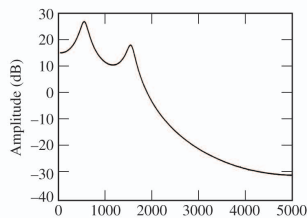
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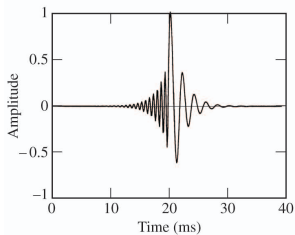
# EXAMPLE OF MINIMUM AND MIXED PHASE



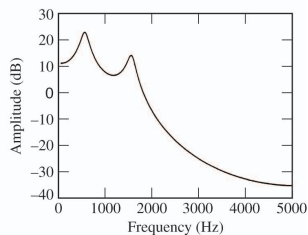
(a)



(b)



(c)



(d)

# FIR AND IIR FILTERS

- FIR:

$$h[n] \neq 0, \quad 0 \leq n < M$$

- IIR:

$$h[n] = \sum_{k=1}^{N_i} A_k c_k^n u[n]$$



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# DISCRETE FOURIER TRANSFORM

Discrete Fourier Transform, DFT, pair:

- Direct:

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}, \quad 0 \leq k \leq N-1$$

- Inverse:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} kn} \quad 0 \leq n \leq N-1$$

Parseval theorem:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

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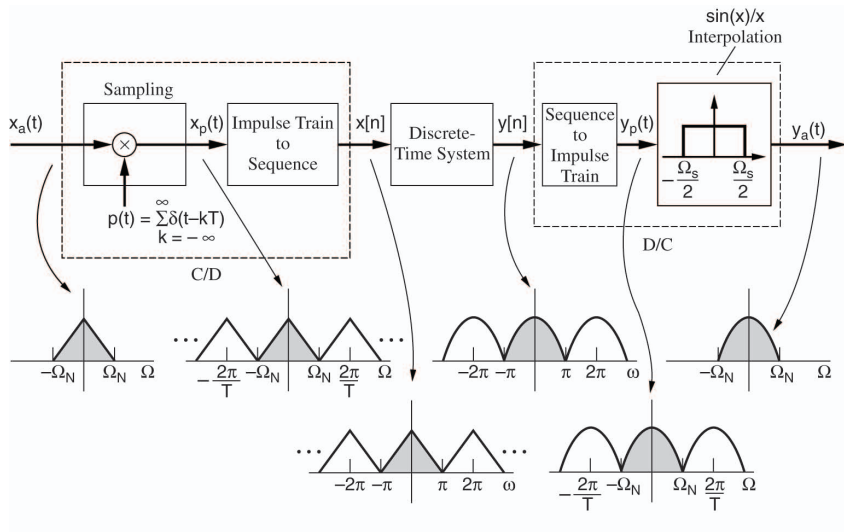
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# ANALOG TO DIGITAL AND DIGITAL TO ANALOG





# Τέλος Ενότητας



Ευρωπαϊκή Ένωση  
Πρωτεύουσα Βρυξελλών



# Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στα πλαίσια του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Κρήτης**» έχει χρηματοδοτήσει μόνο τη αναδιαμόρφωση του εκπαιδευτικού υλικού.
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# Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

# Σημείωμα Χρήσης Έργων Τρίτων

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

## **Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες**

Εικόνες/σχήματα/διαγράμματα/φωτογραφίες που περιέχονται σε αυτό το αρχείο προέρχονται από το βιβλίο:

Τίτλος: *Discrete-time Speech Signal Processing: Principles and Practice*

Prentice-Hall signal processing series, ISSN 1050-2769

Συγγραφέας: Thomas F. Quatieri

Εκδότης: Prentice Hall PTR, 2002

ISBN: 013242942X, 9780132429429

Μέγεθος: 781 σελίδες

και αναπαράγονται μετά από άδεια του εκδότη.