Adaptive Sinusoidal Modeling

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SPCC 2015

Sinusoidal Models

- Sinusoidal Model:

$$x(t) = \sum_{k=-K}^{K} \gamma_k e^{j2\pi f_k t}$$

- Special case, Harmonic Model:

$$x(t) = \sum_{k=-K}^{K} \gamma_k e^{j2\pi k f_0 t}$$

- Estimation of parameters (Linear approach):

$$\gamma = \mathcal{F}_{f_k} \mathbf{s}$$

- Model Evaluation, Mean-Squared Error (MSE):

$$\epsilon = \int_w |s(t) - x(t)|^2 \, dt$$

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iQHM, aQHM, eaQHM, aHM

Towards Adaptive Sinusoidal Models

Estimating Sinusoidal parameters

- Sinusoidal representation for a speech/signal frame:

$$x(t) = \left(\sum_{k=-K}^{K} a_k e^{j2\pi f_k t} \right) w(t)$$

- Methods:
  - FFT-based methods (i.e., QIFFT [Abe et al., 2004–05, [2] [3]])
  - Subspace methods
  - Least Squares (LS) method
- Frequency mismatch:

$$\hat{f}_k = f_k + \eta_k$$

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Quasi-Harmonic Model, QHM [6]

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A few details of QHM

- QHM in the frequency domain:

$$X_k(f) = a_k W(f - \hat{f}_k) + j\frac{b_k}{2\pi} W'(f - \hat{f}_k)$$

- **Decomposition of** b_k: $b_k = \rho_{1,k} a_k + \rho_{2,k} j a_k$
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$$X_k(f) = a_k \left[W(f - \hat{f}_k) - \frac{\rho_{2,k}}{2\pi} W'(f - \hat{f}_k) + j\frac{\rho_{1,k}}{2\pi} W'(f - \hat{f}_k) \right]$$

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- **Approximation of** the *kth* component of QHM

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- Back to the time-domain:

$$x_k(t) \approx a_k \left[e^{j(2\pi\hat{f}_k + \rho_{2,k})t} + \rho_{1,k} t e^{j2\pi\hat{f}_k t} \right] w(t)$$

- Initially, we assumed:

$$x_k(t) = a_k \left[e^{j(2\pi(\hat{f}_k + \eta_k))t} \right] w(t)$$

- in other words, **it is suggested**:

$$\hat{\eta}_k = \rho_{2,k}/2\pi$$

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CONSTRAINTS

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Influence of the Window

- We would like small value for $W''(f)$ at f_k
- $W''(f)$ is influenced by the length, T, of the analysis window, i.e., for rectangular window: $W''(f) \propto T^3$
- So ... we would like a **short analysis window**
- But ... **long analysis window** provides robustness
- Length of the window $\rightleftarrows$ bandwidth

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Influence of frequency mismatch

- Assume

$$y(t) = \alpha e^{j(2\pi \hat{f} t + \eta t)}$$

- modeled by QHM as

$$x(t) = (a + tb)e^{j(2\pi \hat{f} t)} \quad -T \leq t \leq T$$

- LS solution provides:

$$a = \alpha \frac{sin(\eta T)}{\eta T}$$

$$b = \alpha \ 3j(\frac{sin(\eta T)}{\eta^2 T^3} - \frac{cos(\eta T)}{\eta T^2})$$

- Frequency mismatch estimate:

$$\hat{\eta} = 3 \left(\frac{1}{\eta T^2} - \frac{cot(\eta T)}{T} \right)$$

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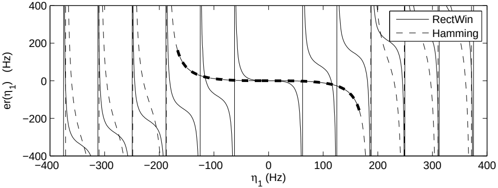
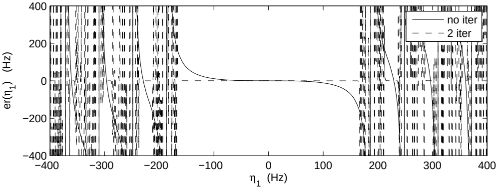
Combining Influences



(a)



(b)

- Iteratively, the bias can be removed when $|\eta| < B/3$, where B is the bandwidth of the squared analysis window.

Robustness against additive noise

- Signal contaminated by noise:

$$y(t) = \sum_{k=1}^{4} a_k e^{j2\pi f_k} + v(t)$$

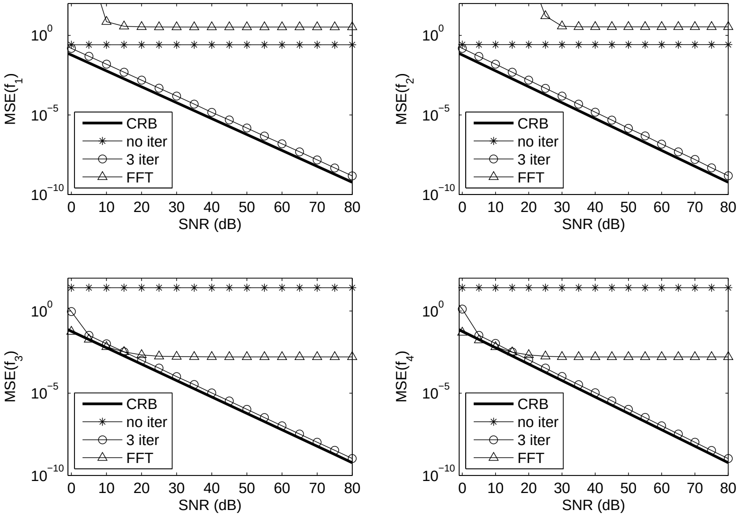
- Mean Squared Error (MSE):

$$MSE\{\hat{f}_k\} = \frac{1}{M}\sum_{i=1}^{M} |\hat{f}_k(i) - f_k|^2$$

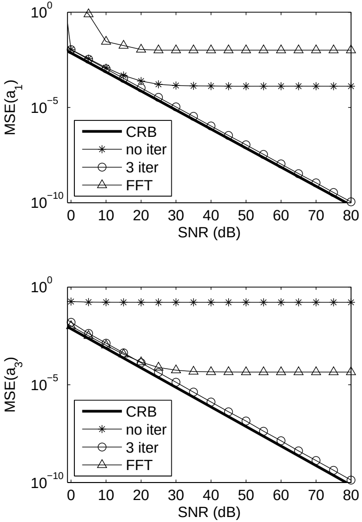
$$MSE\{\hat{a}_k\} = \frac{1}{M}\sum_{i=1}^{M} |\hat{a}_k(i) - a_k|^2$$

- Comparison with Cramer-Rao Bounds (CRB) and QIFFT (Abe et al. 2004)
- 10000 Monte Carlo simulations

MSE of frequencies as a function of SNR.



MSE of amplitudes as a function of SNR.



Notes on QHM

- The approximation made in QHM is valid provided that the frequency mismatch lies in a specific interval.
- This interval is a function of the bandwidth of the analysis window.
- Robustness of QHM against noise was tested and verified.
- It can be shown that iterative QHM is equivalent to (an approximate) Gauss-Newton method.
- There are also relations between QHM and Reassigned Spectrum

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QHM and Gauss-Newton method

- Assuming we have N samples of $x(t) = ce^{j\omega t}w(t)$
- Approximate Gauss-Newton solution for frequency

$$\omega^{(1)} = \omega^{(0)} - \mathcal{R}\left\{ \frac{\sum_{t=-N}^{N} tw^2(t)x(t)e^{-j\omega^{(0)}t}}{c^{(0)}\sum_{t=-N}^{N} t^2 w^2(t)} \right\}$$

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with

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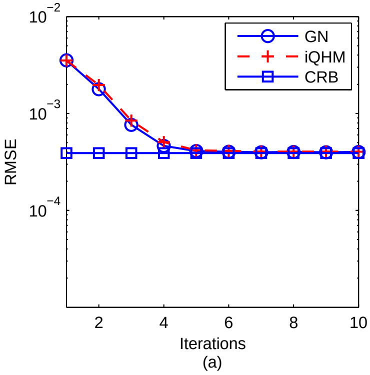
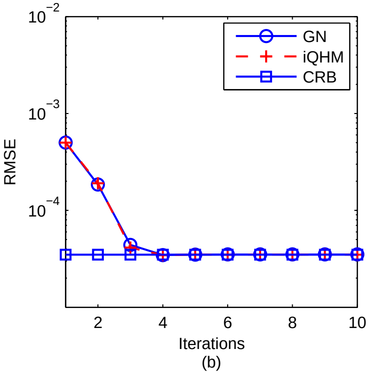
Evaluation: RMSE for frequency

▷ $SNR = 0$dB, $N = 100$ (left), $N = 500$ (right).
CRB: Cramer-Rao lower bound for frequency estimation





Evaluation: RMSE for amplitude

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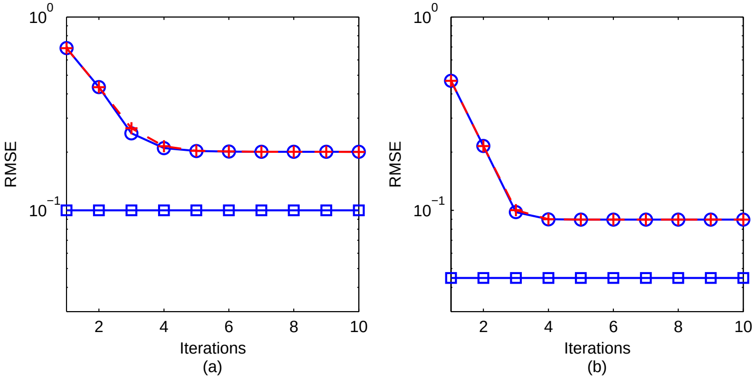

Reassigned Spectrum

- Time relocation:

$$\hat{\tau} = -\frac{\partial \phi(\tau, \omega)}{\partial \omega} = \tau - \Re\left(\frac{X_{Tw}(\tau, \omega)}{X(\tau, \omega)}\right)$$

where

$$X_{Tw}(\tau, \omega) = -\sum_{t=-N}^{N} t w(t) x(t+\tau) e^{-j\omega(t+\tau)}$$

- Frequency relocation:

$$\hat{\omega} = \omega + \frac{\partial \phi(\tau, \omega)}{\partial \tau} = \omega + \Im\left(\frac{X_{\mathcal{D}w}(\tau, \omega)}{X(\tau, \omega)}\right)$$

where

$$X_{\mathcal{D}w}(\tau, \omega) = -\sum_{t=-N}^{N} \frac{dw(t)}{dt} x(t+\tau) e^{-j\omega(t+\tau)}$$

Reassigned Spectrum and QHM

- QHM: $x(t) = (a + tb)e^{j\omega t}w(t)$ with $b = \rho_1 a + \rho_2 j a_1$
- Then, it can be shown:

$$\hat{\tau} = \tau + \frac{W_2}{W_0}\rho_1$$

and (in case of using Gaussian windows):

$$\hat{\omega} = \omega + \frac{W_2}{W_0}\rho_2$$

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$$W_k = \sum_{t=-N}^{N} t^k w(t)$$

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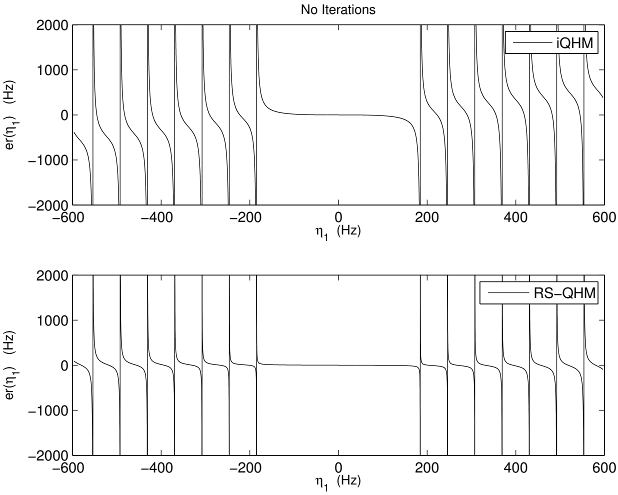
Frequency mismatch estimation error



From QHM to Adaptive QHM, aQHM [7]

- QHM (**stationarity assumption**):

$$x(t) = \left(\sum_{k=-K}^{K} (a_k + tb_k) e^{2\pi j f_k t} \right) w(t)$$

- Adaptive QHM (aQHM):

$$x(t) = \left(\sum_{k=-K}^{K} (a_k + tb_k) e^{j\tilde{\phi}_k(t)} \right) w(t)$$

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$$\tilde{\phi}_k(t) = 2\pi \int_0^t f_k(s) ds + \varphi, \;\; t \in [0, T]$$

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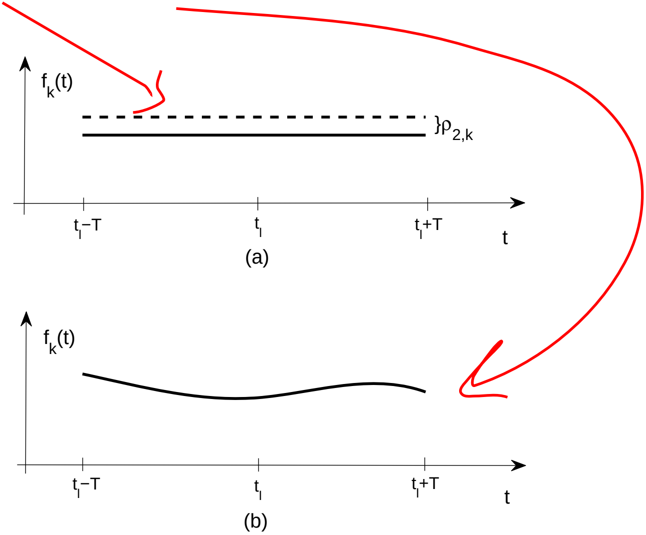
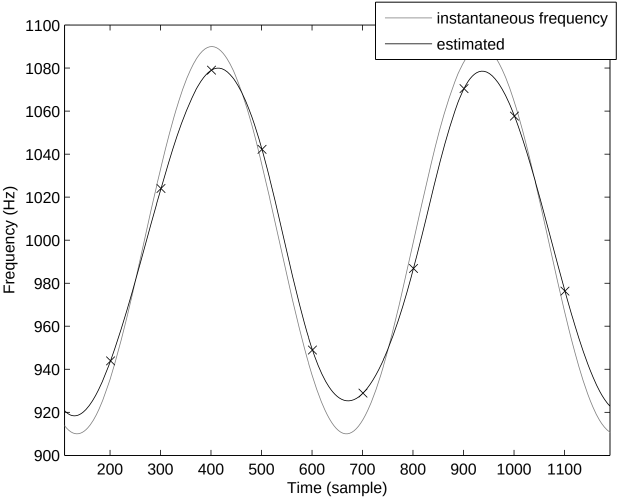
From QHM to aQHM; Graphically



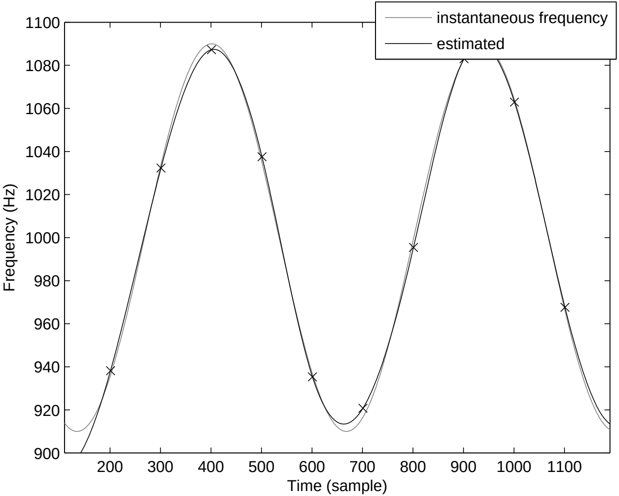
Frame Rate in aQHM

- One sample: no interpolation between estimations
- Higher rates (i.e., 5ms, 10ms): Interpolation between estimates is required:
  - Amplitudes are linearly interpolated
  - Frequencies are interpolated with splines
  - Phases are interpolated by integration of instantaneous frequency

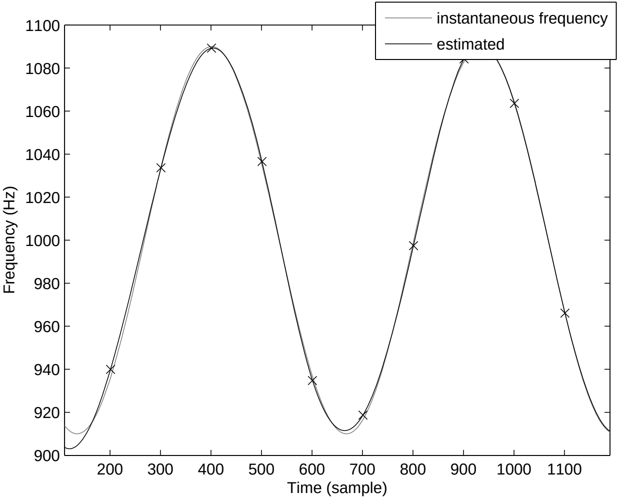
Example of estimation in aQHM: no iteration (QHM)



Example of estimation in aQHM: one iteration



Example of estimation in aQHM: two iterations



Validation

- Let us consider as example:

$$x(t) = (11 - 340t + 4000t^2)e^{j2\pi(280t+19500t^2)}$$

- and frequency mismatch: $|f_k - \zeta_k| = 35Hz$, using Hamming window
- Consider simplified approaches: Sinusoidal-based analysis (SM)
- Mean Absolute Error (MAE) [frame rate: one sample]

	AM	FM (Hz)
QHM, 10ms	0.46	2.15
aQHM (1)	0.008	0.006
SM, 10ms	0.44	3.08

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QHM REMINDER

- QHM:

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- Instantaneous parameters:

$$\begin{aligned} A_k(t) &= \sqrt{(a_k^R + tb_k^R)^2 + (a_k^I + tb_k^I)^2} \\ \Phi_k(t) &= 2\pi f_k t + atan \frac{a_k^I + tb_k^I}{a_k^R + tb_k^R} \\ F_k(t) &= f_k + \frac{1}{2\pi} \frac{a_k^R b_k^I - a_k^I b_k^R}{M_k^2(t)} = f_k + \frac{1}{2\pi} \rho_{2,k} \end{aligned}$$

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QHM and AM-FM decomposition

- AM-FM signal

$$y(t) = \sum_{k=1}^{K(t)} a_k(t) cos(\phi_k(t)),$$

- Taylor series expansion of the instantaneous phase of kth component:

$$\phi_k(t) = 2\pi\zeta_k t + \sum_{i=0}^{\infty} \phi_{k,i} \frac{t^i}{i!}$$

- Instantaneous frequency of the kth component at $t = 0$:

$$\nu_k(0) = \zeta_k + \frac{\phi_{k,1}}{2\pi}$$

- ... and previously we had:

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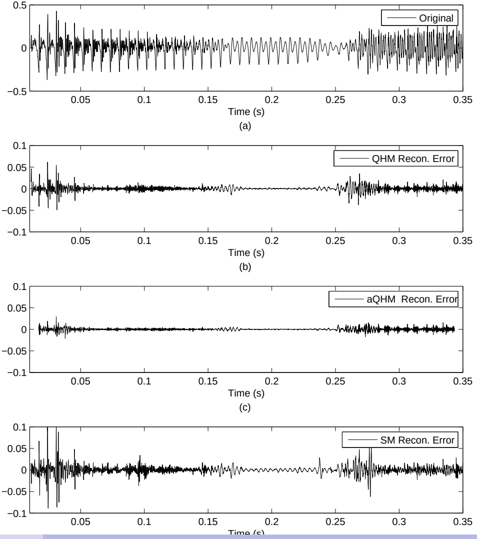
Reconstruction errors with QHM, aQHM, SM



aQHM: AM-FM decomposition of Voiced Speech

- Signal Reconstruction

$$\hat{x}(t) = \sum_{k=1}^{K} \hat{A}_k(t) \cos\left(\hat{\Phi}_k(t)\right)$$

- Test on 5 minutes of 20 female and male voiced speech (TIMIT)
- Average Signal-to-Error Reconstruction Ratio (in dB)

	Male	Female
QHM	23.9	29.1
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Extended aQHM

- Recall aQHM:

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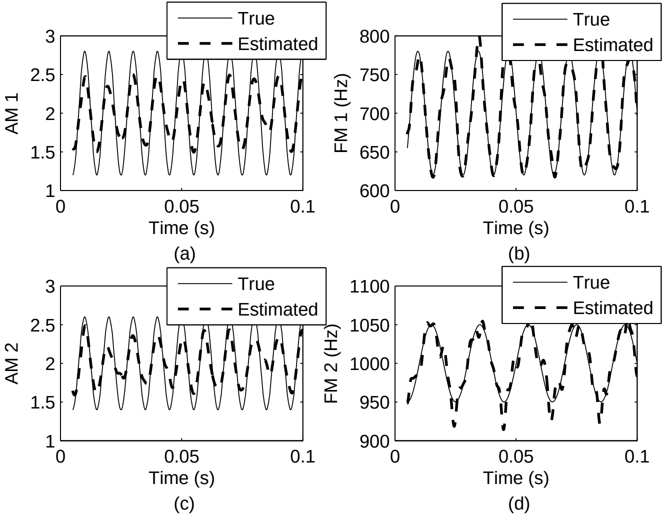
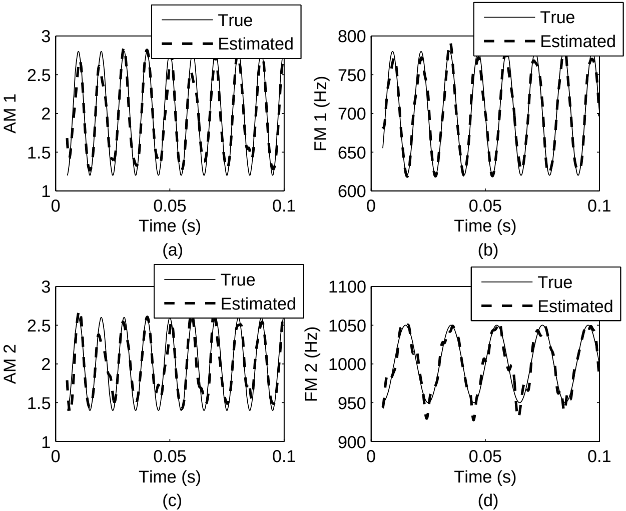
- Extended aQHM:

$$x(t) = \sum_{k=-K}^{K} (a_k + tb_k) \alpha(t) e^{j\tilde{\phi}_k(t)}$$

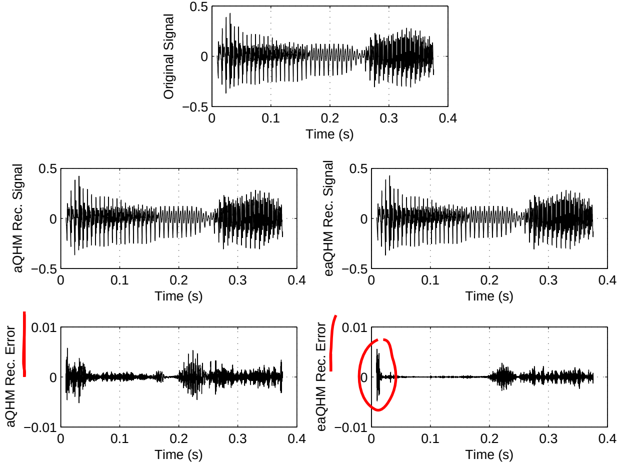
AM-FM modeling: aQHM



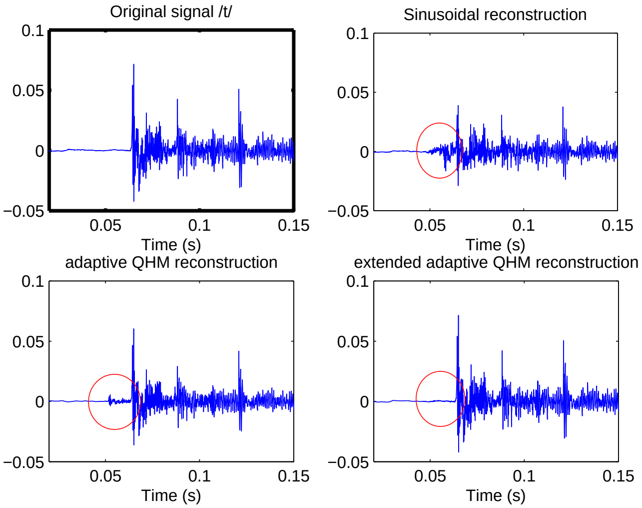
AM-FM modeling: extended aQHM



Comparing Adaptive Models



Stop modeling



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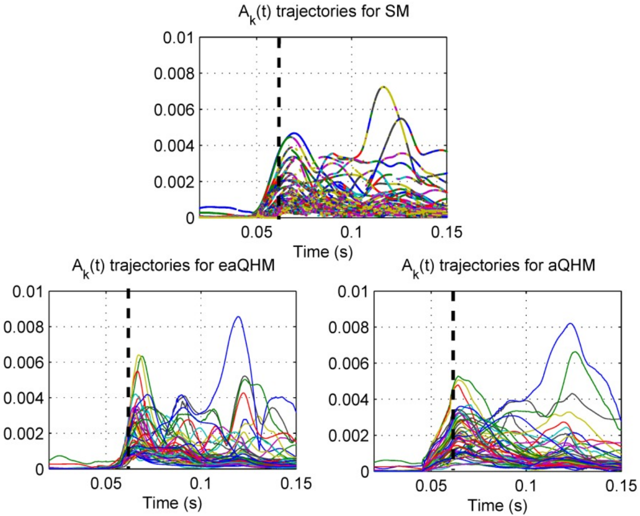

Large scale evaluation

Global Signal to Reconstruction Error Ratio (dB)						
Model	/*p*/	/*t*/	/*k*/	/*b*/	/*d*/	/*g*/
SM	12.7	12.8	12.4	16.6	14.9	15.3
aQHM	19.9	20.6	21.7	28.3	26.9	27.5
eaQHM	25.4	25.7	27.2	32.9	32.2	32.9

Speech Modifications

- High quality pitch scale modification
- Within glottal cycle formant tracking and modifications towards voice conversion
- Free pre-echo effect in time scaling
- Emotion detection and control

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- Statistical modeling: *naturally* smooth trajectories, articulators tracking, adaptation and controllability, modulations
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- Data driven dynamic information
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Key papers to read

- R. McAulay and T. Quatieri: *Speech analysis/Synthesis based on a sinusoidal representation* IEEE TASSP, Vol.34, No.4, Aug 1986, pp 744-754.
- Y. Pantazis, O. Rosec, and Y. Stylianou: *Adaptive AM-FM Signal Decomposition with Application to Speech Analysis* IEEE TASLP, Vol.19, No.2, Feb 2011, pp 290-300.
- Y. Pantazis, O. Rosec, and Y. Stylianou: *Iterative Estimation of Sinusoidal Signal Parameters*. IEEE SPL, Vol.17, No.5, May 2010, pp. 461-464
- P. Babu and P. Stoica: *Comments on Iterative Estimation of Sinusoidal Signal Parameters.* Comments on IEEE SPL, pp.1022-1023, Vol.17, No.12, Dec 2010
- Y. Pantazis, O. Rosec, and Y. Stylianou: *Reply to "Comments on 'Iterative Estimation of Sinusoidal Signal Parameters' "* IEEE SPL, pp. 1024-1026, Vol.17, No.12, Dec 2010
- G. Degottex and Y. Stylianou: *Analysis and Synthesis of Speech using an Adaptive Full-band Harmonic Model* IEEE TASLP, Vol.21, Issue 10, pp 2085-2095, 2013

T. F. Quatieri, *Discrete-Time Speech Signal Processing*.
Engewood Cliffs, NJ: Prentice Hall, 2002.

M. Abe and J. S. III, "CQIFFT: Correcting Bias in a Sinusoidal Parameter Estimator based on Quadratic Interpolation of FFT Magnitude Peaks," Tech. Rep. STAN-M-117, Stanford University, California, Oct 2004.

M. Abe and J. S. III, "AM/FM Estimation for Time-varying Sinusoidal Modeling," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, (Philadelphia), pp. III 201–204, 2005.

J. Laroche, "A new analysis/synthesis system of musical signals using Prony's method. Application to heavily damped percussive sounds.," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, (Glasgow, UK), pp. 2053–2056, May 1989.

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Acknowledgments

- My colleagues: Yannis Pantazis, Olivier Rosec (Voxygen), Vassilis Tsiaras, Gilles Degottex
- My ex-PhD Student: Giorgos Kafentzis
- Tom Quatieri and Prentice Hall for gave me the permission to use figures from Tom's book[1]

THANK YOU

for your attention on this part as well!