Appendix C. A solution of example cancer diagnostic problem using MC-SVMs

Consider an example diagnostic problem with three possible outcomes: T-cell acute lympboblastic leukemia (ALL), B-cell ALL, and acute myelogenous leukemia (AML), using expression levels of two genes: c-Myb and SNF2. Given a training set of patients (see **Figure C1**), our goal is to build a multicategory linear SVM diagnostic model that can be later applied to new patients. Although this example is highly simplified for clarity, the ideas generalize to all kinds of classification problems and data characteristics.

One-vs-rest (OVR). Application of the three binary OVR classifiers (**Figure C2**) yields a decision surface divided by three separate hyperplanes (dashed lines). The shaded regions in the figure correspond to tie situations when two or none classifiers are active (i.e. vote) at the same time (**Table C1**).

Consider classification of a new sample (triangular shaped in the figure) in the ambiguous region 5. This sample receives votes from both AML and ALL T-cell classifiers, however its distance from the "ALL T-cell vs. ALL B-cell and AML" hyperplane is larger than one from "AML vs. ALL B-cell and ALL T-cell" hyperplane. Hence, this sample belongs to ALL T-cell class.

The combined OVR decision function separates the decision surface by a solid bold line (**Figure C2**). Notice that the final decision function differs significantly from the original one which corresponded to the solution of *k* QP optimization problems.

One-vs-one (OVO). Using the OVO technique, a decision surface is divided by three separate hyperplanes (dashed lines) obtained by binary SVMs corresponding to one versus one decisions (**Figure C3**). The application of *Max Wins strategy* (**Table C2**) results in division of decision surface into three regions (separated by bold dashed lines) and the small shaded ambiguous region in the middle of the figure. The tie-breaking strategy applied to the ambiguous region produces the final decision function depicted with solid bold lines and bold dashed lines. Notice that in this example the final decision function does not differ significantly from the initial one corresponding to the solution of $\binom{k}{2}$ optimization problems.

DAGSVM. The application of DAGSVM involves training of $\binom{k}{2}$ binary OVO classifiers. For the testing phase, we follow the DDAG shown in the **Figure C4**. We note that there is no unique subdivision of the decision surface for this MC-SVM technique. Depending on the path in the DDAG (**Figure C4**), the decision surface is divided by two out of three dashed lines (i.e. binary one-versus-one classifiers) in the **Figure C3**.

Method by Weston and Watkins (WW). An example of the WW decision surface obtained for the diagnostic problem example is shown in the **Figure C5**. Note that all the classes are separated with the maximum margin.

Method by Crammer and Singer (CS). The decision surface for the CS method roughly corresponds to one for the WW, although the methods differ algorithmically (**Figure C5**).

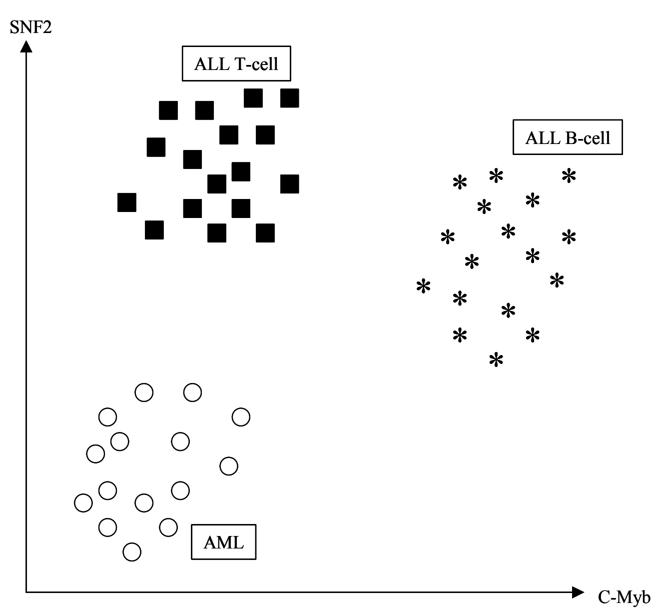

Figure C1. *Example diagnostic problem with three outcomes: ALL T-cell (■), ALL B-cell (✱), and AML (○).*

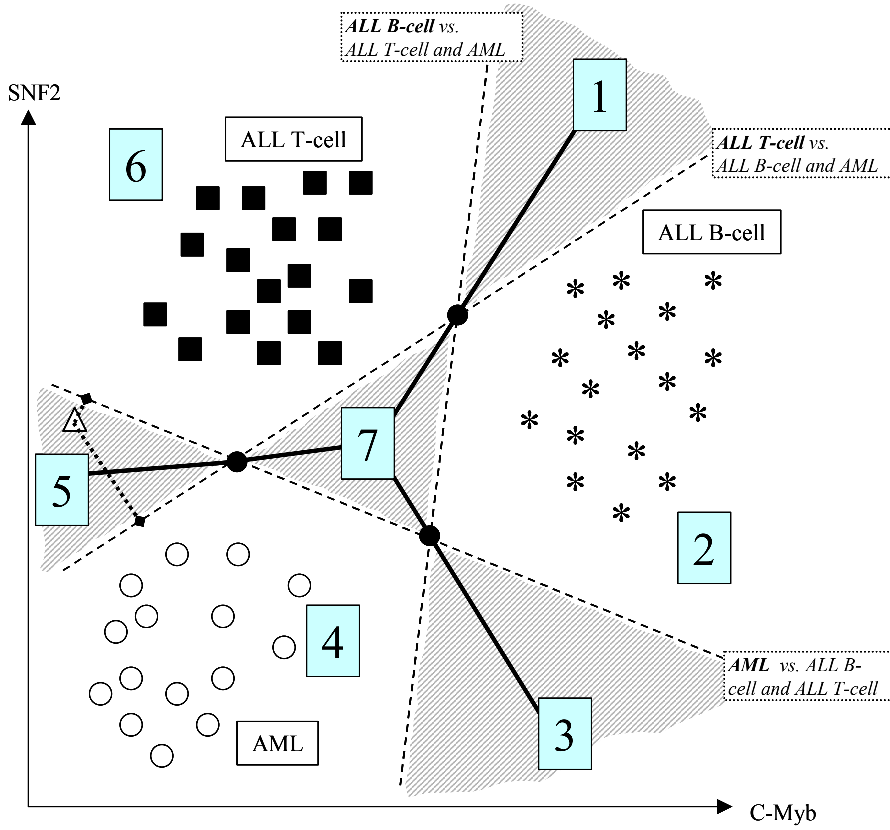

Figure C2. *OVR MC-SVM is applied to the example diagnostic problem with three outcomes: ALL T-cell (■), ALL B-cell (✱), and AML (○).*

Region	**Decision of the classifer**			**Resulting class**
	AML vs. (ALL B-cell and ALL T-cell)	ALL T-cell vs. (ALL B-cell and AML)	ALL B-cell vs. (ALL T-cell and AML)	
1	-	*ALL T-cell*	*ALL B-cell*	*?*
2	-	-	*ALL B-cell*	*ALL B-cell*
3	*AML*	-	*ALL B-cell*	*?*
4	*AML*	-	-	*AML*
5	*AML*	*ALL T-cell*	-	*?*
6	-	*ALL T-cell*	-	*ALL T-cell*
7	-	-	-	*?*

Table C1. *Three binary OVR classifiers are applied to the example diagnostic problem (see **Figure C2**). The column "Resulting class" contains the resulting classification of each region. Cells with "?" correspond to tie situations when two or none classifiers are active (i.e. vote) at the same time. Please see text for how ties are resolved.*

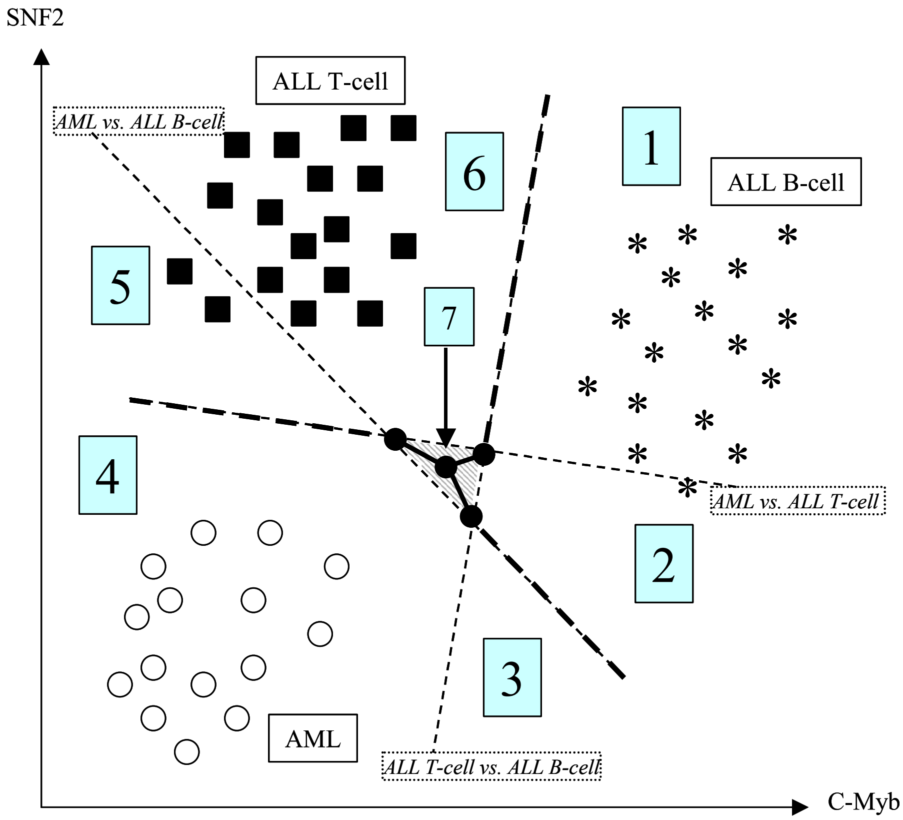

Figure C3. *OVO MC-SVM is applied to the example diagnostic problem with three outcomes: ALL T-cell (■), ALL B-cell (✱), and AML (○).*

Region	**Decision of the classifier**			**Resulting class**
	AML vs. ALL B-cell	ALL T-cell vs. ALL B-cell	AML vs. ALL T-cell	
1	***ALL B-cell***	***ALL B-cell***	*ALL T-cell*	*ALL B-cell*
2	***ALL B-cell***	***ALL B-cell***	*AML*	*ALL B-cell*
3	***AML***	*ALL B-cell*	***AML***	*AML*
4	***AML***	*ALL T-cell*	***AML***	*AML*
5	*AML*	***ALL T-cell***	***ALL T-cell***	*ALL T-cell*
6	*ALL B-cell*	***ALL T-cell***	***ALL T-cell***	*ALL T-cell*
7	*ALL B-cell*	*ALL T-cell*	*AML*	*?*

Table C2. *Three binary OVO classifiers are applied to the example diagnostic problem (see **Figure C3**). The column "Resulting class" contains the resulting classification of each region according to "Max Wins" strategy. Cell with "?" corresponds to tie situation when three classifiers vote at the same time for different classes). Please see text for how ties are resolved.*

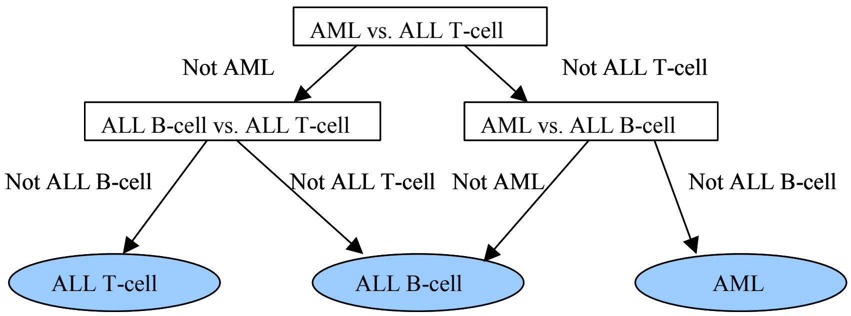

Figure C4. *DAGSVM algorithm: rooted binary decision directed tree (DDAG) for testing of unseen samples for the example diagnostic problem.*

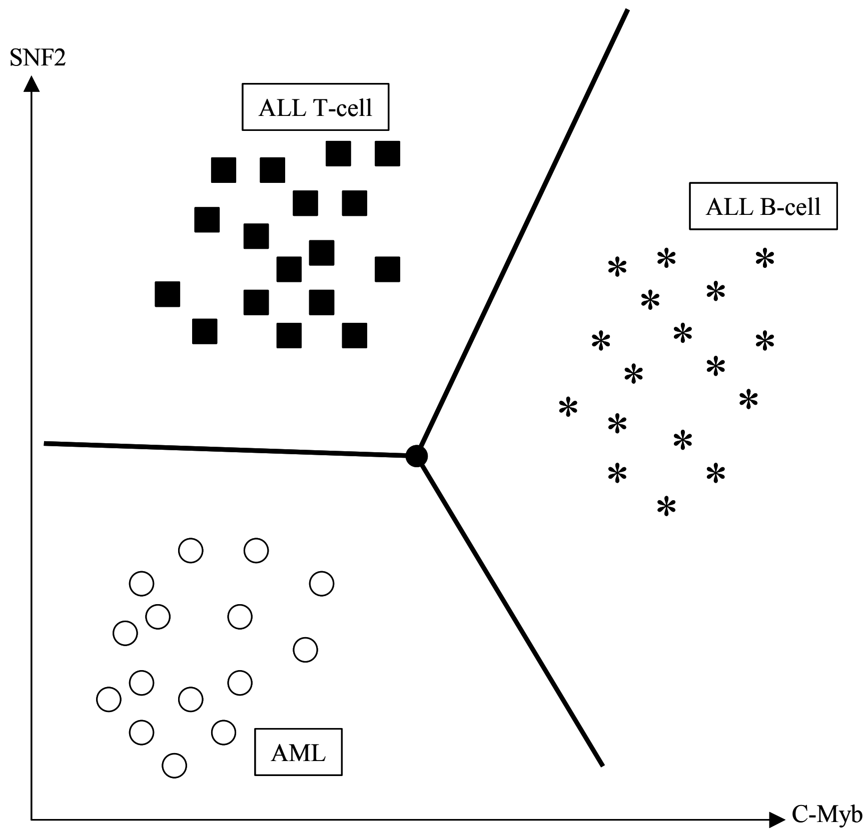

Figure C5. *WW MC-SVM is applied to the example diagnostic problem with three outcomes: ALL T-cell (■), ALL B-cell (✻), and AML (○).*